Lecture 8.

Summary

1. Maxwell equations

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{D} = \varepsilon \vec{E}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J} \qquad \vec{B} = \mu \vec{H}$$

$$\nabla \cdot \vec{D} = \rho_{net} \qquad \vec{J} = \sigma \vec{E}$$

$$\nabla \cdot \vec{B} = 0$$

$$\vec{E} \perp \vec{H} \perp \vec{k}$$

2. Plane wave solution

$$\vec{E}_c = \vec{E}_{co} \exp[i(\omega t - \vec{k} \cdot \vec{r})]$$

$$\vec{k} \cdot \vec{k} = \frac{\omega^2}{N^2}$$

$$N = \sqrt{\tilde{\varepsilon}_r} = n + i\kappa$$

3. Poynting vector

$$\langle \vec{S} \rangle = \frac{1}{2} \mathrm{Re}(\vec{E}_c \times \vec{H}_c^*)$$

4. Boundary conditions

$$\vec{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$$

$$\vec{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0$$

$$\vec{n} \times (\vec{E}_1 - \vec{E}_2) = 0$$

$$\vec{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$$

$$\vec{E}_i = \vec{E}_{i\parallel} \exp[-i(\omega t - \vec{k}_i \cdot \vec{r})]$$

plane wave

$$= \vec{E}_{i\parallel} \exp\{-i[\omega t - k_{xi}x - k_{yi}y - k_{zi}z]\} \quad (k_{xi}^2 + k_{yi}^2 + k_{zi}^2 = \frac{\omega^2}{c^2})$$

($k_{yi} = 0$)

$$\vec{E}_r = \vec{E}_{r\parallel} \exp\{-i[\omega t - k_{xr}x - k_{yr}y - k_{zr}z]\}$$

($k_{yr} = 0$)

$$\vec{E}_t = \vec{E}_{t\parallel} \exp\{-i[\omega t - k_{xt}x - k_{yt}y - k_{zt}z]\}$$

($k_{yt} = 0$)

No surface charge , $z=0$

$k_{xi} = k_1 \sin\theta_i$, $k_{xr} = k_1 \sin\theta_r$, $k_{xt} = k_2 \sin\theta_t$

$$E_{i\parallel} \cos\theta_i \exp[+k_{xi}x] + E_{r\parallel} \cos\theta_r \exp[+k_{xr}x] = E_{t\parallel} \exp[+k_{xt}x]$$

$$k_1 \sin\theta_i = k_1 \sin\theta_r = k_2 \sin\theta_t$$

$$\theta_i = \theta_r = \theta_1 \quad , \quad \theta_t = \theta_2$$

$$\frac{\omega}{c_1} \sin\theta_1 = \frac{\omega}{c_2} \sin\theta_2$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \rightarrow \text{Snell law}$$

$$E_{i\parallel} \cos\theta_i + E_{r\parallel} \cos\theta_r = E_{t\parallel} \cos\theta_t \qquad (1)$$

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{ix} & 0 & E_{iz} \end{pmatrix} = -\mu \begin{pmatrix} -i\omega H_{x0} \\ -i\omega H_{y0} \\ -i\omega H_{z0} \end{pmatrix}$$

$E_{ix} = E_{\parallel} \cos\theta \exp[\ ]$, $E_{iz} = -E_{\parallel} \sin\theta \exp[\ ]$

$$H_{y0} = \frac{i k E_{\parallel i}}{i\omega\mu} \Rightarrow \vec{H}_{yi} = \frac{n_1 E_{\parallel}^{a}}{\mu c} \exp\{-i(\omega t - k_{xi}x - k_{zi}z)\}$$

$$\vec{H}_{yr} = -\frac{n_1 E_{r\parallel}}{\mu c} \exp\{-i(\omega - k_{xr}x + k_{zi}z)\}$$

Continuity of tangential component of $\vec{H} \Rightarrow$

$$n_1 E_{i\parallel} - n_1 E_{r\parallel} = n_2 E_{t\parallel}$$

$$r_{\parallel} = \frac{E_{\parallel r}}{E_{\parallel i}} = \frac{-n_2 \cos\theta_i + n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t}$$

$$t_{\parallel} = \frac{2 n_1 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t}$$

similarly

$r_{\perp}$

$t_{\perp}$

Poynting Vector : $\langle\vec{S}\rangle = \frac{1}{2} \mathrm{Re}\{\vec{E} \times \vec{H}^*\}$

~~$\langle \vec{S}_i \rangle = \frac{1}{2\mu c_0} n_1 |E_{\parallel i}|^2$~~

~~Reflectivity~~ $= \frac{1}{2} \mathrm{Re} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ E_{\parallel i}\cos\theta_i \exp(ik_x x) & 0 & -E_{\parallel i}\sin\theta_i \exp(+ik_x x) \\ 0 & +\frac{n_1 E_{\parallel i}^*}{\mu c_0}\exp(-ik_x x) & 0 \end{vmatrix}$

$$= \frac{1}{2} \mathrm{Re}\left\{ \frac{n_1 E_{\parallel}^2 \sin\theta_i}{\mu c_0} \hat{i} + \frac{n_1 E_{\parallel}^2 \cos\theta_i}{\mu c_0} \hat{k} \right\}$$

~~Bi-directional reflecte~~

Reflectivity $\quad R = \left|\frac{E_{\parallel r}}{E_{\parallel i}}\right|^2 = |r|^2$

Transmissivity $\quad T = \frac{S_{zt}}{S_{zi}} = \frac{n_2 \cos\theta_2}{n_1 \cos\theta_1} |t|^2 \Rightarrow R + T = 1$

~~$|k_2| = \ldots$~~

$$k_2 = \frac{N_2 \omega}{c_0}$$

~~$\vec{E}_t = \vec{E}_{t\parallel} \exp\{-i\omega(t \ldots)\}$~~

$$k_2 \sin\theta_t = k_1 \sin\theta_1$$

$$N_2 \sin\theta_t = n_1 \sin\theta_1$$

θ_t complex. $\qquad \sin\theta_t = \dfrac{e^{i\theta_t} - e^{-i\theta_t}}{2i}$

~~$\cos\theta_t$~~ $\qquad \cos\theta_t = \dfrac{e^{i\theta_t} + e^{-i\theta_t}}{2} = a + bi$

$$\vec{E}_t = \vec{E}_{t\parallel} \exp\{-i(\omega t - \underbrace{k_2 \sin\theta_t}_{\text{real}} x - k \cos\theta_t z)\}$$

$$= \vec{E}_{t\parallel} \exp\{-i(\omega t - az \underbrace{)}_{-k_2 \sin\theta_t x} - bz\}$$

constant phase plane

$$\omega t - az - k_2 \sin\theta_t x = 0$$

constant amplitue plane $\quad z = \text{constant}. \quad \Rightarrow$ Inhomgenen wave

Fresnel formula $r_\parallel \rightarrow$ complex

$t_\parallel \rightarrow$ complex.

But Poynting vector

$$\langle \hat{S} \rangle = \frac{1}{2} \mathrm{Re} \left\{ \frac{N_2 E_\parallel^2 \sin\theta_2}{\mu c_0} \hat{i} + \frac{N_2 E_{\parallel t}^2 \cos\theta_t}{\mu c_0} \hat{k} \right\}$$

$$R = |r_\parallel|^2 \quad , T = \frac{\mathrm{Re}(N_2 \cos\theta_t)}{n_1 \cos\theta_i} |t_\parallel|^2.$$

Discussion: (1) Critical angle

$$\theta_2 = 90^\circ$$

$$n_1 \sin\theta_{cr} = n_2 \Rightarrow \sin\theta_{cr} = \frac{n_2}{n_1}$$

when $n_1 > n_2$

(2) when $\theta_1 > \theta_{cr}$,

Evanescent wave

$$\vec{E}_t = \vec{E}_{t\parallel} \exp[i(\omega t - k_{xt} x - k_{zt} z)]$$

$$k_{zt} = k_t \cos\theta_2 = \frac{\omega}{c}\sqrt{1-\sin^2\theta_2}$$

$$= \frac{\omega}{c}\sqrt{1-\left(\frac{n_1 \sin\theta_1}{n_2}\right)^2} = +i\kappa$$

$$\vec{E}_t = \vec{E}_{t\parallel} \exp[-\kappa z]$$

(3) Brewster angle

$$r_\parallel = 0 \qquad \begin{cases} n_1 \cos\theta_2 = n_2 \cos\theta_1 \\ n_1 \sin\theta_1 = n_2 \sin\theta_2 \end{cases}$$

$$\Rightarrow \tan\theta_1 = \frac{n_2}{n_1} \quad \text{— Brewster angle}$$

only $r_\perp \neq 0$,

(4) If media 2 is absorbing