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Nonlinear Dynamics I: Chaos

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Contents

0 Acknowledgements and references

Since 1989 I have taught a one-semester course on nonlinear dynamics and chaos to MIT undergraduates. Students come from thoughout the institute, but predominantly from mathematics, physics, and my own department (Earth, Atmospheric, and Planetary Sciences). Since 1998 the course has been jointly sponsored by the Department of Mathematics.

The lecture notes that follow have undergone frequent revisions, but perhaps never more so than in 2004, when they were (finally) converted to LaTeX. The conversion followed from LaTeX notes prepared by Kurt Steinkraus, a student in 2003.

Advice and assistance from my TA's has not only been much appreciated, but frequently resulted in new ideas for the course. The TA's included, in rough order of appearance, Andrew Gunstensen, John Olson, Olav van Genabeek, Kelvin Chan, Davide Stelitano, Joshua Weitz, Alison Cohen, Greg Lawson, and David Forney. Thanks are of course also due to the students themselves.

The construction of the course has been influenced heavily by the book by Bergé, Pomeau, and Vidal [1]. Although some sections of the notes, particularly in the first half of the course, derive directly from their book, a greater debt manifests itself in a shared "philosophy" and organization: the introduction by way of oscillators, the emphasis on data analysis, the extensive comparison between theory and experiment, and, perhaps most importantly, the progression, in the second half of the course, from partial differential equations to ordinary differential equations to maps.

There are many other sources of inspiration. Occasionally these are listed explicitly in the notes, or references to figures contained within them are made. Particular lectures and references used to construct the notes (aside from Bergé, Pomeau, and Vidal [1]) include the following:

- Systems of ODE's: Strogatz [2] and Beltrami [3].
- Liouville's theorem: Tolman [4].
- Van der Pol equation: Strogatz [2].
- Floquet theory: Bender and Orszag [5].
- Mathieu equation: Landau and Lifshitz [6].
- Fluid dynamics: Tritton [7], Landau and Lifshitz [8], and Palm [9].
- Strange attractors: Abraham and Shaw [10].
- Lorenz equations: Lorenz [11] and Tritton [7].
- Hénon attrator: Hénon [12].
- Fractals: Barnsley [13] and Grassberger and Procaccia [14].
- Period doubling: Feigenbaum [15], Schuster [16], and Kadanoff [17].

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