Some Relationships for Gases

define some units

$kJ := 10^3\cdot J$

$kmol := 10^3 mole$

these are extracted from Van Wylen & Sonntag, Fudamentals of Classical Thermodynamics, Third Edition to which page numbers and equation numbers apply
2006: included reference to text: Woud section 2.23 in [W n.nn]

section 3.4 Equations of state for the vapor phase of a simple compressile substance - page 41 (Woud page 20)

gas at low density (experiment) $p\cdot\bar{v} = \bar{R}\cdot T$ $\bar{R} = universal\_gas\_constant$ (3.1)

$\overline{xxx} = mole\_basis$

$R\_bar := 8.3144 \frac{kJ}{kmol\cdot K}$ units sometimes ... $\frac{kJ}{kg\_mol\cdot K}$

$R = \frac{\bar{R}}{mw}$ $mw = molecular\_weight$ for R_bar above $mw = \frac{kg}{kmol}$

$p\cdot V = m\cdot R\cdot T$ or ... $\boxed{p\cdot v = R\cdot T}$ (3.2) $\boxed{\frac{p_1\cdot v_1}{T_1} = \frac{p_2\cdot v_2}{T_2}}$ (3.5) [W 2.32, 2.33[

section 4.3 Work done at moveable boundary of simple compressible system - page 63

if ... $p\cdot V^n = constant$ $n = 1$ $W_{1\_2} = \int_{V_1}^{V_2} p\,dV = p_1\cdot V_1\cdot\int_{V_1}^{V_2}\frac{1}{V}\,dV = p_1\cdot V_1\cdot\ln\left(\frac{V_2}{V_1}\right)$ (4.5)

section 5.6 The Constant-Volume and Constant-Pressure Specific Heats - page 98

specific heat = increment of heat Q to change T by 1 deg $c = \frac{1}{m}\cdot\frac{\delta Q}{\delta T}$ $\frac{1}{m} = specfic$

two cases: 1) constant volume $c_v = \frac{1}{m}\cdot\frac{\delta Q}{\delta T}$ constant volume

1st law ... $\delta Q = dE + \delta W = dU + dKE + dPE + \delta W$ (5.4)

$\delta Q = dU + p\cdot\delta V$ $dKE = dPE = 0$ $\delta W = p\cdot\delta V = 0$

$c_v = \frac{1}{m}\cdot\frac{\delta Q}{\delta T} = \frac{1}{m}\cdot\frac{\delta U}{\delta T} = \frac{\delta u}{\delta T}$ $\boxed{c_v = \frac{\delta u}{\delta T}}$ constant volume (5.14) [W 2.36]

2) constant pressure $\delta Q = dU + p\cdot\delta V = \delta H$ as .... $dH = d(U + p\cdot V) = dU + p\cdot dV + V\cdot dp$ $dp = 0$

$c_p = \frac{1}{m}\cdot\frac{\delta Q}{\delta T} = \frac{1}{m}\cdot\frac{\delta H}{\delta T} = \frac{\delta h}{\delta T}$ $\boxed{c_p = \frac{\delta h}{\delta T}}$ constant pressure (5.15) [W 2.37]

section 5.7 The Internal Energy, Enthalpy and Specific Heats of Ideal Gases - page 100

ideal gas $p \cdot v = R \cdot T$ $u = f(T)$ experiment (Joule)

$c_v = \frac{\delta u}{\delta T}$ u not a function of v => $\boxed{du = c_{vo} \cdot dT}$ vo => constant volume ideal gas (5.20)

also ... $h = u + p \cdot v = u(T) + R \cdot T = h(T)$ i.e. h = F(T) only

$c_p = \frac{\delta h}{\delta T}$ => $\boxed{dh = c_{po} \cdot dT}$ po => constant pressure ideal gas (5.24)

relation between c_{vo} and c_{po} ... $h = u + pv = u + R \cdot T$ $\frac{dh}{dT} = \frac{du}{dT} + R$ differentiate w.r.t T

$c_{po} = c_{vo} + R$ or ... $\boxed{c_{po} - c_{vo} = R}$ (5.27)

with constant c $\boxed{h_2 - h_1 = c_{po} \cdot (T_2 - T_1)}$ otherwise integrate if c(T) known or tables (5.29) [W 2.38]

section 7.10 Entropy Change of an Ideal Gas - page 206

$T \cdot ds = du + p \cdot dv$ (7.7) [W 2.18]

$du = c_{vo} \cdot dT$ and ... $p \cdot v = R \cdot T$ => $p = R \cdot \frac{T}{v}$ => $T \cdot ds = c_{vo} \cdot dT + R \cdot \frac{T}{v} \cdot dv$

$ds = c_{vo} \cdot \frac{dT}{T} + R \cdot \frac{dv}{v}$ (7.19) $\boxed{s_2 - s_1 = c_{vo} \cdot \ln\left(\frac{T_2}{T_1}\right) + R \cdot \ln\left(\frac{v_2}{v_1}\right)}$ (7.24) $c_{vo} = \text{constant}$

otherwise integrate or use tables

$T \cdot ds = dh - v \cdot dp$ (7.7) [W 2.21]

$dh = c_{po} \cdot dT$ and ... $p \cdot v = R \cdot T$ => $v = R \cdot \frac{T}{p}$ => $T \cdot ds = c_{po} \cdot dT - R \cdot \frac{T}{p} \cdot dp$

$ds = c_{po} \cdot \frac{dT}{T} - R \cdot \frac{dp}{p}$ (7.21) => $\boxed{s_2 - s_1 = c_{po} \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)}$ (7.23) $c_{po} = \text{constant}$

otherwise integrate or use tables

page 211 introduce specific heat ratio γ $\boxed{\gamma = \frac{c_{po}}{c_{vo}}}$ (7.30) [W 2.44] $\gamma = \gamma(T)$ as ... $c = c(T)$

from above $c_{po} - c_{vo} = R$ $c_{vo} = c_{po} - R = \gamma \cdot c_{vo} - R$ => $c_{vo} \cdot (1 - \gamma) = -R$

changing signs ... $\boxed{c_{vo} = \frac{R}{\gamma - 1}}$ (7.31)

similarly ... $c_{po} - c_{vo} = R$ $c_{po} = R + c_{vo} = R + \frac{c_{po}}{\gamma}$ => $c_{po} \cdot \left(1 - \frac{1}{\gamma}\right) = R = c_{po} \cdot \frac{\gamma - 1}{\gamma}$

$$\boxed{c_{po} = R \cdot \frac{\gamma}{\gamma - 1}}$$ (7.31)

for ***constant*** specific heat = perfect gas

reversible, adiabatic process ... $T \cdot ds = du + p \cdot dv = 0$ reversible, adiabatic ...

$du + p \cdot dv = c_{vo} \cdot dT + p \cdot dv$ $p \cdot v = R \cdot T$ => $dT = \frac{1}{R} \cdot (p \cdot dv + v \cdot dp)$

$$0 = c_{vo} \cdot dT + p \cdot dv = \frac{c_{vo}}{R} \cdot (p \cdot dv + v \cdot dp) + p \cdot dv = \frac{c_{vo}}{R} \cdot (p \cdot dv + v \cdot dp) + p \cdot dv = \frac{R}{\gamma - 1} \cdot \frac{1}{R} \cdot (p \cdot dv + v \cdot dp) + p \cdot dv$$

=> $(p \cdot dv + v \cdot dp) + p \cdot dv(\gamma - 1) = v \cdot dp + \gamma \cdot p \cdot dv = 0$ => $\frac{dp}{p} + \gamma \cdot \frac{dv}{v} = 0$ dividing by pv

integrating ... $\ln(p) + \gamma \cdot \ln(v) = \text{constant}$ or ... $\boxed{p \cdot v^{\gamma} = \text{constant}}$ (7.32) [W 2.49]

rearranging ... $\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma}$

for ideal gas $\frac{p_1 \cdot v_1}{T_1} = \frac{p_2 \cdot v_2}{T_2}$ => $\frac{v_1}{v_2} = \frac{p_2}{T_2} \cdot \frac{T_1}{p_1} = \frac{T_1}{T_2} \cdot \frac{p_2}{p_1}$ =>

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = \left(\frac{T_1}{T_2} \cdot \frac{p_2}{p_1}\right)^{\gamma} = \left(\frac{T_1}{T_2}\right)^{\gamma} \cdot \left(\frac{p_2}{p_1}\right)^{\gamma} \quad \Rightarrow \quad \left(\frac{p_2}{p_1}\right)^{1-\gamma} = \left(\frac{T_1}{T_2}\right)^{\gamma} \qquad \frac{p_2}{p_1} = \left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{1-\gamma}} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

and ... for reversible adiabatic process constant specific heat (ideal gas)

$\boxed{\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}}$ (7.35) or ... $\boxed{T \cdot p^{\frac{\gamma}{\gamma-1}} = T \cdot p^{\frac{1-\gamma}{\gamma}} = \text{constant}}$ [W 2.47]

$\boxed{T \cdot v^{\gamma-1} = \text{constant}}$ [W 2.48]

to explore the effect of temperature on the coefficients of specific heat the following is provided

to calculate enthalpies at non-standard conditions one approach is to use constant-pressure specific heats from Table A.9 of Van Wylen and Sonntag ... The applicable temperature range for these materials is 300 - 3500 deg K with less than approximately 0.5 % maximum error to experimental values. see also figure 5.10 on page 103.

$\theta = \frac{T}{100}$ T in deg K

O2 $C_{po\_O2}(\theta) := 37.432 + 0.020102\cdot\theta^{1.5} - 178.57\cdot\theta^{-1.5} + 236.88\cdot\theta^{-2}\ \frac{kJ}{kmol\cdot K}$

C_O2 $C_{po\_C\_O2}(\theta) := -3.7357 + 30.529\cdot\theta^{0.5} - 4.1034\cdot\theta + 0.024198\cdot\theta^{2}\ \frac{kJ}{kmol\cdot K}$

given in kJ/kmole*K so divide by molecular weight to get kJ/kg*K

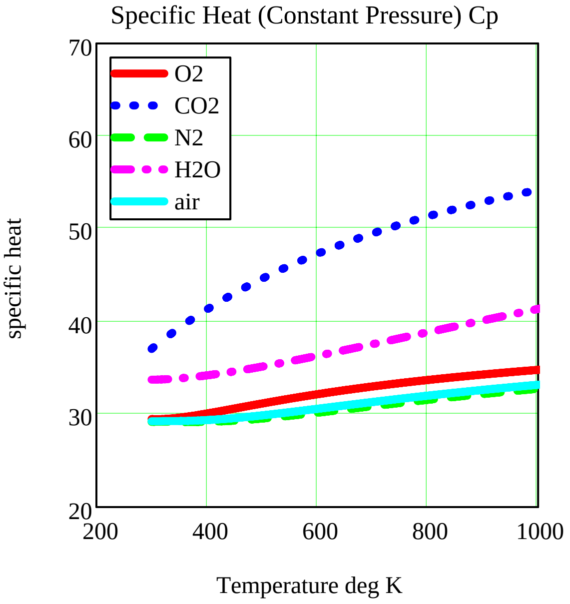
N2 $C_{po\_N2}(\theta) := 39.060 - 512.79\cdot\theta^{-1.5} + 1072.7\cdot\theta^{-2} - 820.40\cdot\theta^{-3}\ \frac{kJ}{kmol\cdot K}$

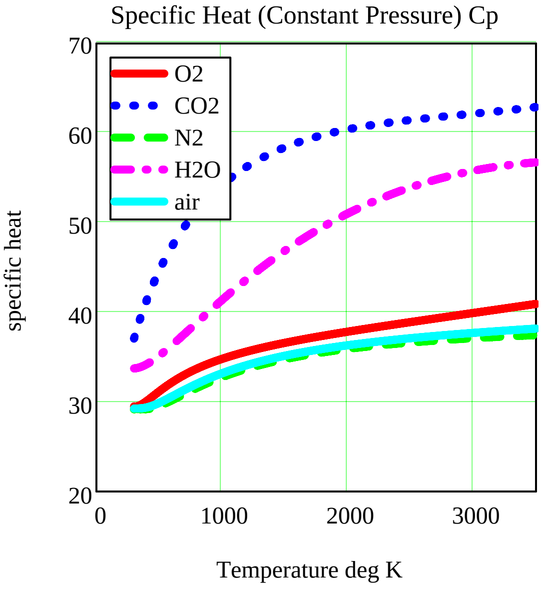
H2_O $C_{po\_H2\_0}(\theta) := 143.05 - 183.54\cdot\theta^{0.25} + 82.751\cdot\theta^{0.5} - 3.6989\cdot\theta\ \frac{kJ}{kmol\cdot K}$

air based on volumetric average as these are on mole basis ...

air $C_{po\_air}(\theta) := 0.21\cdot C_{po\_O2}(\theta) + 0.79\cdot C_{po\_N2}(\theta)$

$T := 300 .. 3500$





relatively constant at reasonably low temperatures the next section was added Nov 2005 to organize plots for Brayton and dual (Seiliger) cycles

applications of above relationships to processes

$R := 0.287$ $\gamma := 1.4$ $c_{po} := 1.0035 \quad \frac{kJ}{kg \cdot K}$ $c_{vo} := \frac{c_{po}}{\gamma}$ $c_{vo} = 0.717$

units assumed are p = bar, v = m^3/kg, T = K, s = kJ/kg*K
100 necesary for consistency in numerical calculations without units.

isentropic adiabatic compression (expansion) ___________________________________ *pressure ratio known*

$p_{initial} := 1$ $T_{initial} := 400$ $s_{initial} := 1$ $v_{initial} := \frac{R \cdot T_{initial}}{p_{initial} \cdot 100}$ $v_{initial} = 1.148$

$p_{final} := 10$ $T_{final} := T_{initial} \cdot \left(\frac{p_{final}}{p_{initial}}\right)^{\frac{\gamma-1}{\gamma}}$ $s_{final} := s_{initial}$ $v_{final} := v_{initial} \cdot \left(\frac{p_{initial}}{p_{final}}\right)^{\frac{1}{\gamma}}$

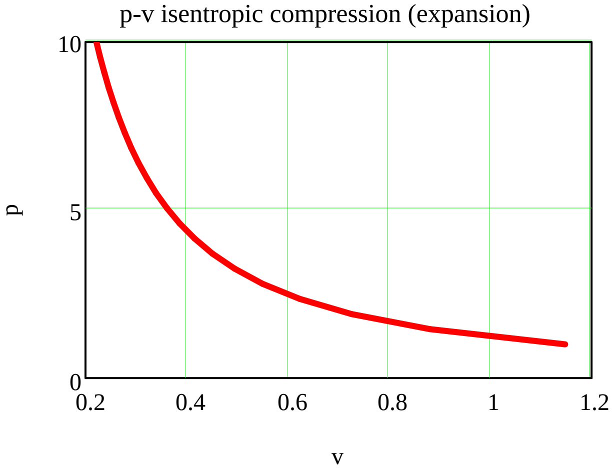
$T_{final} = 772.279$ $s_{final} = 1$ $v_{final} = 0.222$

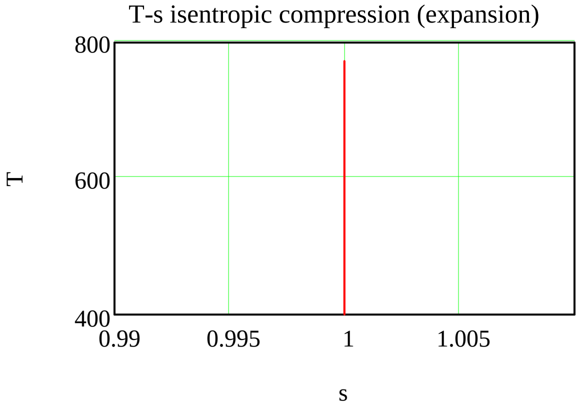
p - v plot parameterized on either p or v say p

$\delta p := \frac{p_{final} - p_{initial}}{20}$ to plot 20 points and accomodate increases and decreases

$pp := p_{initial}, p_{initial} + \delta p \,..\, p_{final}$ T - s is straight line, need 2 points

$v\_plot(pp) := v_{initial} \cdot \left(\frac{p_{initial}}{pp}\right)^{\frac{1}{\gamma}}$ $Ts\_plot := \begin{pmatrix} T_{initial} & s_{initial} \\ T_{final} & s_{final} \end{pmatrix}$ $Ts\_plot = \begin{pmatrix} 400 & 1 \\ 772.279 & 1 \end{pmatrix}$





isentropic adiabatic compression (expansion)
volume ratio known

$v_{initial} := 1$ $T_{initial} := 400$ $s_{initial} := 1$ $p_{initial} := \frac{R\cdot T_{initial}}{v_{initial}\cdot 100}$ $p_{initial} = 1.148$

$v_{final} := \frac{1}{10}$ $T_{final} := T_{initial}\cdot\left(\frac{v_{initial}}{v_{final}}\right)^{\gamma-1}$ $s_{final} := s_{initial}$ $p_{final} := p_{initial}\cdot\left(\frac{v_{initial}}{v_{final}}\right)^{\gamma}$

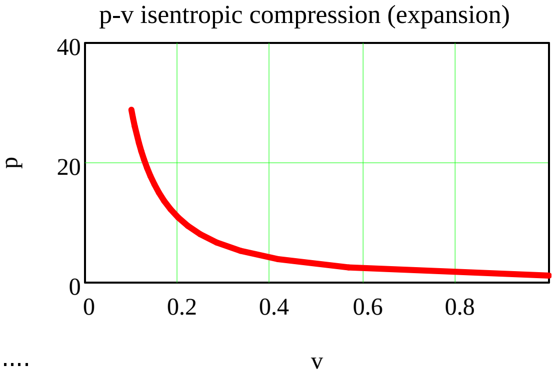
$T_{final} = 1.005\times 10^{3}$ $s_{final} = 1$ $p_{final} = 28.836$

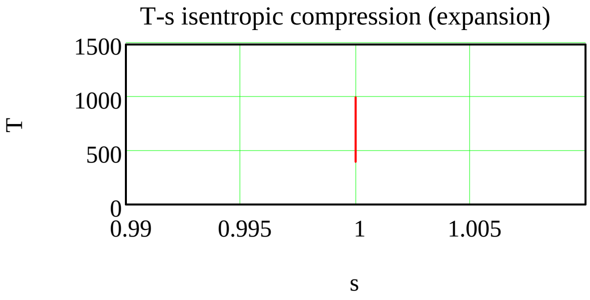
p - v plot parameterized on either p or v say p

$\delta p := \frac{p_{final} - p_{initial}}{20}$ to plot 20 points and accomodate increases and decreases

$pp := p_{initial}, p_{initial} + \delta p \,..\, p_{final}$ T - s is straight line, need 2 points

$pv\_plot(pp) := v_{initial}\cdot\left(\frac{p_{initial}}{pp}\right)^{\frac{1}{\gamma}}$ $Ts\_plot := \begin{pmatrix} T_{initial} & s_{initial} \\ T_{final} & s_{final} \end{pmatrix}$ $Ts\_plot = \begin{pmatrix} 400 & 1 \\ 1.005\times 10^{3} & 1 \end{pmatrix}$

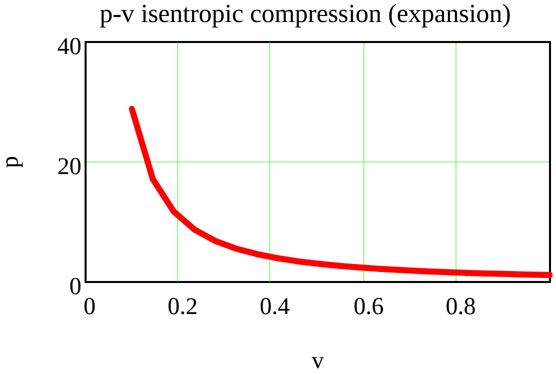




or ....

$\delta v := \frac{v_{final} - v_{initial}}{20}$ to plot 20 points and accomodate increases and decreases

$vv := v_{initial}, v_{initial} + \delta v \,..\, v_{final}$ $pv\_plot(vv) := p_{initial}\cdot\left(\frac{v_{initial}}{vv}\right)^{\gamma}$



same plot as above

heat transfer at constant pressure ....

set up to go from $T_{initial}$ to T_{final} at p = constant, $s_{initial}$ assumed = 1

$T_{initial} := 298$ $s_{initial} := 1$ $p_{constant} := 3$ bar $v_{initial} = 1$

$T_{final} := 500$ final end state calculation

$$s_{final} := s_{initial} + c_{po} \cdot \ln\left(\frac{T_{final}}{T_{initial}}\right)$$ $s_{final} = 1.519$ $v_{final} = 0.1$

inbetween states for plotting ....

p - v is a straight line
needing only end
points to plot

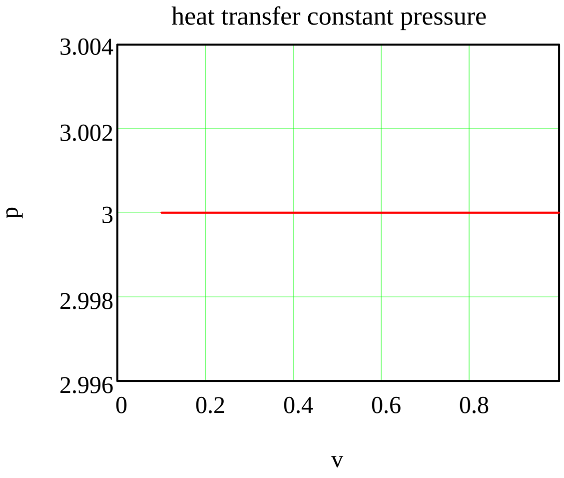
$$pv\_plot := \begin{pmatrix} p_{constant} & v_{initial} \\ p_{constant} & v_{final} \end{pmatrix}$$

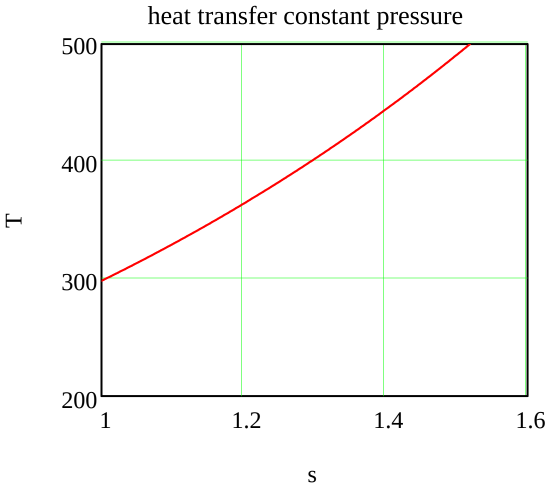
$$pv\_plot = \begin{pmatrix} 3 & 1 \\ 3 & 0.1 \end{pmatrix}$$

if desired for a simulation where interim p v points are required, choose one parameter, e.g. T or v and calculate v or T using pv/T = R for the other

$TT := T_{initial} .. T_{final}$

$$s_{plot}(TT) := \left(s_{initial} + c_{po} \cdot \ln\left(\frac{TT}{T_{initial}}\right)\right)$$





heat transfer at constant volume ....

set up to go from $T_{initial}$ to T_{final} at v = constant, $s_{initial}$ assumed = 1

$T_{initial} := 450$ $\quad s_{initial} := 1$ $\quad v_{constant} := 0.3$ $\quad p_{initial} := \frac{R \cdot T_{initial}}{v_{constant} \cdot 100}$ $\quad p_{initial} = 4.305$

$T_{final} := 298$ final end state calculation

$s_{final} := s_{initial} + c_{vo} \cdot \ln\left(\frac{T_{final}}{T_{initial}}\right)$ $\quad s_{final} = 0.705$ $\quad p_{final} := \frac{R \cdot T_{final}}{v_{constant} \cdot 100}$ $\quad p_{final} = 2.851$

inbetween states for plotting ....

p - v is a straight line needing only end points to plot

if desired for a simulation where interim p v points are required, choose one parameter, e.g. T or v and calculate v or T using pv/T = R for the other

$$pv\_plot := \begin{pmatrix} p_{initial} & v_{constant} \\ p_{final} & v_{constant} \end{pmatrix}$$

$$TT := T_{initial} .. T_{final}$$

$$pv\_plot = \begin{pmatrix} 4.305 & 0.3 \\ 2.851 & 0.3 \end{pmatrix}$$

$$s_{plot}(TT) := \left(s_{initial} + c_{vo} \cdot \ln\left(\frac{TT}{T_{initial}}\right)\right)$$