

## Practice Problems No.2

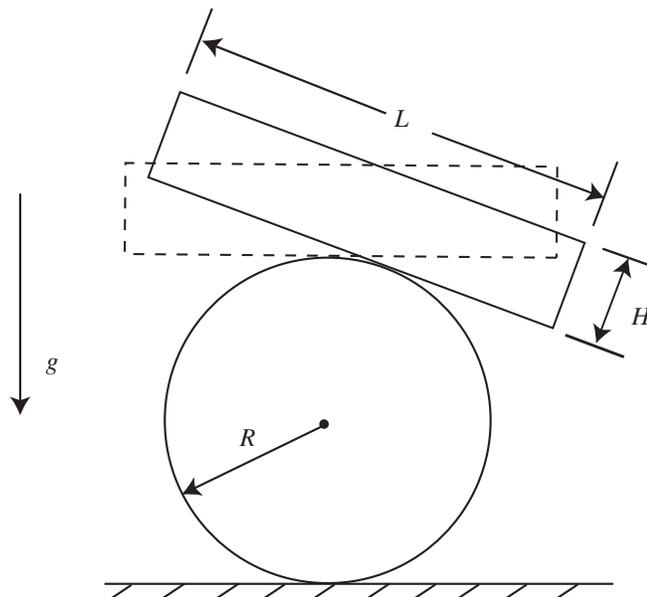
**Problem 1**

A rigid block of height  $H$ , length  $L$  and depth  $D$  rests on a rigid cylinder of mass  $M$  and radius  $R$ , as shown in the sketch. The cylinder rolls on the floor without slipping and the block rolls on the cylinder without slipping as well.

(a) Study the geometry of motion of the two rigid bodies, paying attention to the rolling constraints. Select a suitable set of independent generalized coordinates to describe this motion. Show your choice of coordinates *clearly* on a sketch.

(b) Derive the governing equations of motion in terms of these coordinates.

(c) Examine the stability to small perturbations of the equilibrium position of the block.



### Problem 2

The linkage precesses about the vertical axis at the constant rate  $\Omega$ . The small disk B and slider C each have mass  $m$ , and the mass of each link is negligible. The spring has stiffness  $k$  and its unstretched length is  $2l$ . Identify the two possible constant values of  $\theta$  in the physically meaningful range  $0 \leq \theta < \pi/2$  corresponding to steady precession with  $\dot{\theta} = 0$ . Prove that one of these possibilities is always unstable, while the other is a stable position that exists only if  $k$  is sufficiently large.

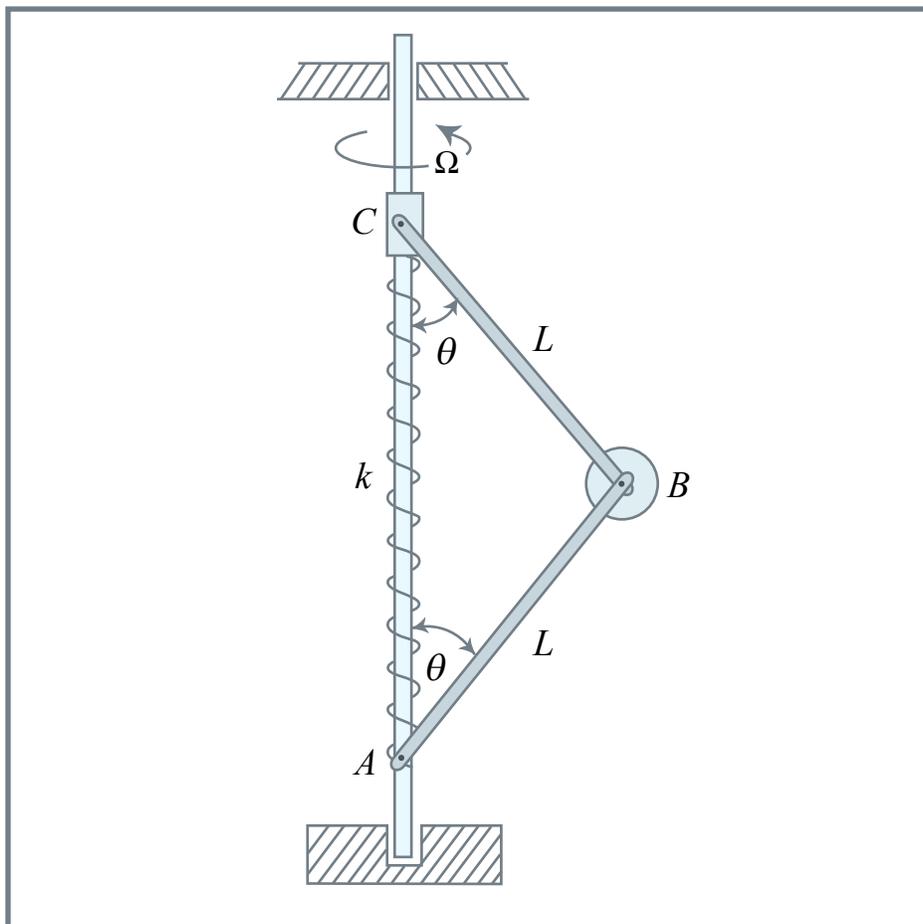


Figure by OCW.

### Problem 3

The shaft of length  $2L$  is forced to precess at the steady rate  $\Omega$  with the fixed angle  $\theta$  shown. On the shaft are two rotors spinning in *opposite* directions with absolute spin  $n$ . The rotors have masses  $m$ , diametral moments of inertia  $I_1$  and axial moments of inertia  $I_3$ . Neglecting gravity, find the torque required to maintain the precession.

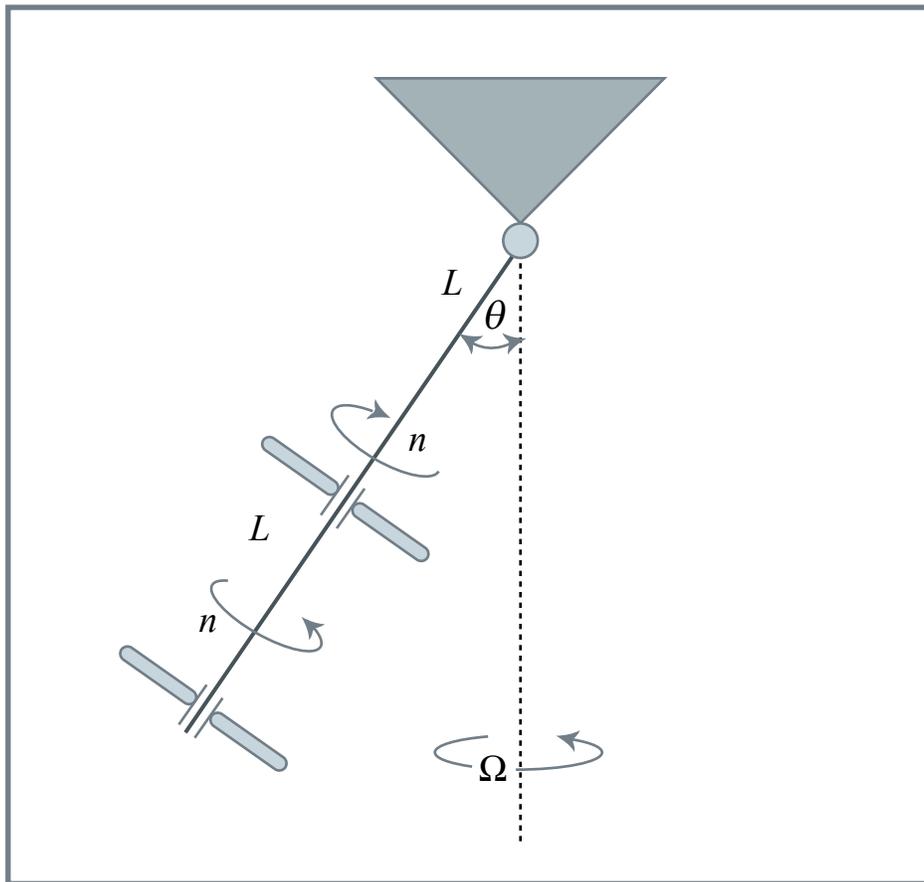
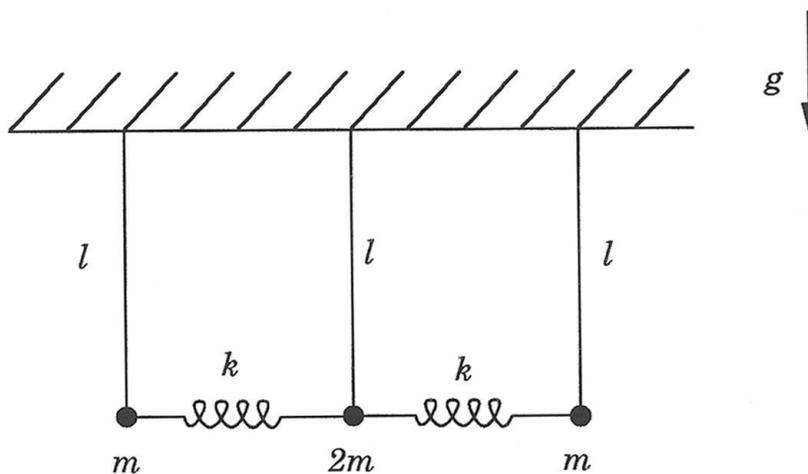


Figure by OCW.

### Problem 4

Three pendulums, all of length  $l$ , are symmetrically arranged as shown in the figure. The center bob, of mass  $2m$ , is connected by ideal springs of force constant  $k$  to the outer bobs, each of mass  $m$ . What are the frequencies and relative displacements of the pendulums when they are vibrating in each of the three normal modes of the system? Consider only vibrations in the plane of the figure. [Caution: a little physical insight will save a lot of algebra in this problem.]



### Problem 5

Consider small motion in a plane about the position of static equilibrium of the given system. Solve for the natural frequencies and the corresponding amplitude ratios.

