2.25 ADVANCED FLUID MECHANICS Fall 2004

FINAL EXAM

Tuesday, December 14, 2004, 1:30 – 4:30 P.M.

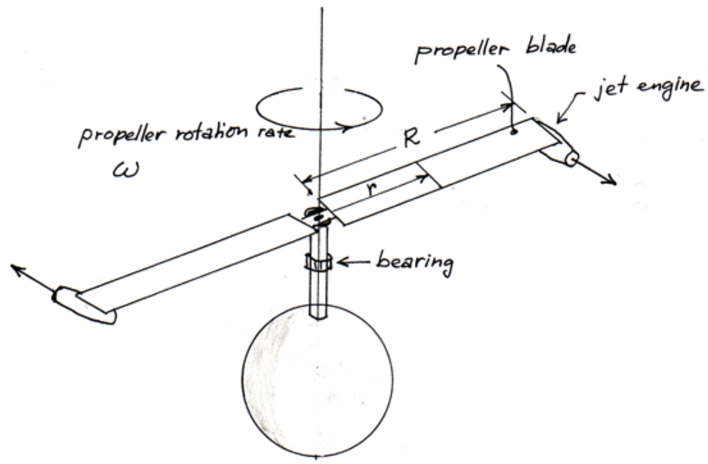
OPEN EXAM WHEN INSTRUCTED AT 1:30 PM

THERE ARE TWO PROBLEMS
OF EQUAL WEIGHT
(each graded out of 20)

Please read each question in its entirety before starting

Please answer each question in a SEPARATE book
Be sure to write your name on each book
No additional loose-leaf pages should be turned in (they will not be graded)

PROBLEM 1 An Unmanned Helicopter



A small unmanned reconnaissance helicopter is powered by jets attached to the tips of its two propeller blades. The idea is that a self-propelled rotor mounted on a torque-free bearing will minimize co-rotation of the helicopter's body, making control (and reconnaissance) easier. In what follows, assume that the spherical body and its contents are *not* set in rotation.

The lift and drag coefficients,

$$c_L = \frac{L'}{\frac{1}{2}\rho V^2 b}$$

and

$$c_D = \frac{D'}{\frac{1}{2}\rho V^2 b}$$

have values that are established by wind tunnel tests. L' is the lift force *per unit length* of propeller blade, D' is the drag force per unit length of propeller blade, ρ is the density of the ambient air, and V denotes the local air velocity incident on rotor blade. b and R are the blade width and blade length, respectively.

Also given are:

- M_0, mass of each jet engine
- m', mass *per unit length* of propeller blade
- M, mass of total helicopter system
- V_0, jet exhaust velocity
- ρ_0, jet exhaust density

Assume that other masses are unimportant.

A. Obtain an equation for the upward lift force provided by the rotor at a **steady** angular rotation rate ω (rad/s). Assume that the blades move through an essentially static fluid, i.e. that the propeller velocity is much higher than any induced angular rotation of the air.

B. At what rotation rate ω_* (rad/s) will the helicopter hover in a stationary state?

C. What jet exit velocity V_0 is required to maintain the rotation rate ω_* in **B**?

NOT REQUIRED FOR EXAM (extra credit):

D. What is the power required for maintaining the rotation rate in **B**? (Not as simple as it may seem at first sight.)

Problem 2: Lubricated Pipelining

A common engineering challenge faced in pumping viscous crude oil over long distances is the large power consumption required to convey the oil through the pipeline. One proposed solution is to lubricate the pipeline as shown below using a thin layer of an immiscible fluid (such as water) with a lower viscosity to surround the oil and lubricate the motion. We shall model the flow as flow in a cylindrical pipe of radius R with a core of thickness R_1 consisting of very viscous liquid oil with viscosity μ_1 surrounded by a shell of water (or other low viscosity fluid) of thickness $\delta = R - R_1$ that is density matched (so that $\rho_1 = \rho_2 = \rho$) with viscosity $\mu_2 < \mu_1$. The interfacial tension between the two liquids is denoted σ. The average velocity of the oil through the pipe is denoted $\overline{v}_o = Q_{oil}/\pi R_1^2$

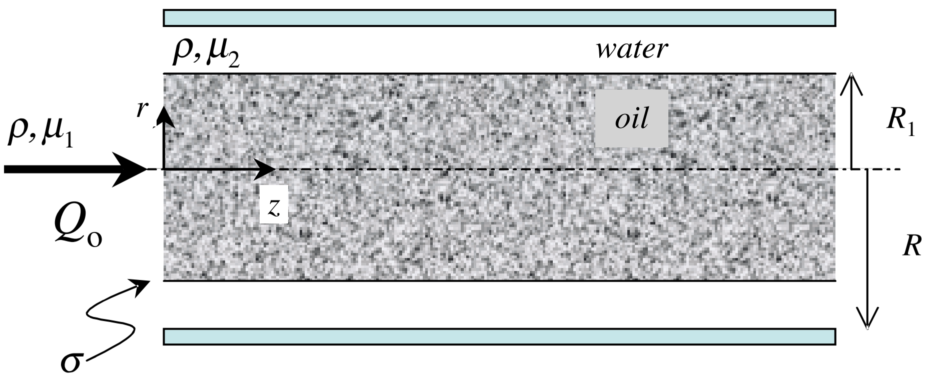
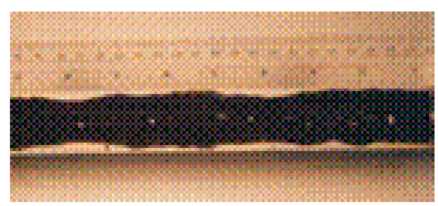


Figure 1: geometry of a lubricated pipeline

a) Although the oil-water interface shown in the figure above is depicted as planar, in reality under certain operating conditions interfacial waves may form as shown in the picture opposite:



http://www.aem.umn.edu/research/pipeline/horizontalindex.html

Use dimensional analysis to determine an appropriate dimensionless form for expressing the fully-developed pressure drop per unit length in the pipe $-\frac{\partial P}{\partial z} = \frac{\Delta P}{L}$ as a function of the other relevant parameters in the problem. Use the average oil velocity $\overline{v}_o = Q_{oil}/\pi R_1^2$ and core radius R_1 as two of your primary variables together with as many other parameters as you need. Which dimensionless group is important in determining whether waves will develop. Based on your physical understanding of interfacial processes, express an appropriate inequality on the range for this dimensionless parameter in order for waves <u>not</u> to form.

b) Assuming that your criterion above is satisfied so that the flow in the pipe remains a perfect smooth core-annular flow as shown in the sketch, write down the appropriate boundary condition for the shear stress on the interface $r = R_1$. Furthermore, provide a criterion under which the change in pressure across the interface is negligible.

c) Use these boundary conditions to find expressions for the fully-developed velocity field $v_z(r)$ that are valid in the core domain $0 \le r \le R_1$ and the shell $R_1 \le r \le$ R. On a single large graph (at least 0.5 page in size), sketch the velocity profile **and** the shear stress profile across the entire pipe (i.e. for the region $0 \le r \le R$).

d) The flow in the pipeline is typically started impulsively by imposing a sudden increase in the pressure gradient along the pipe, and the flow takes a period of time to become fully developed. Draw a large diagram and sketch the shape of the velocity field $v_z(r,t)$ as a function of time. Provide an engineering estimate of the total time taken for the flow field to reach steady state.

e) Find expressions for the volume flow rate of oil Q_o and for the volume flow rate of water Q_w through the pipeline as a function of the imposed pressure ΔP and the other physical parameters defined in the figure.

f) The results of your analysis can be used to optimize the lubricated pipeline operation. For example; consider the viscosities μ_1, μ_2 and density ρ to all be held constant. Show that at any fixed value of the imposed pressure gradient $\Delta P / L$ there is an optimal value of the core radius (denoted R_1^*) that maximizes the volume flow rate of oil through the pipe. Derive an expression for R_1^* and explain (very briefly) why this occurs.

NOT REQUIRED FOR EXAM (extra credit):

g) If the outer layer of fluid becomes very thin ($\delta << R$) and of very low (but still non-zero!) viscosity ($\mu_2 << \mu_1$) then a number of simplifying approximations can be made in the governing equations. Show that in this limit the inner fluid can have a large velocity very close to the wall which makes it *appear* to 'slip' at the wall (i.e. at $r = R_1 \approx R$) with a slip velocity v_w that is proportional to the wall shear stress. Find the coefficient of proportionality.

Navier Stokes equation in $\{r, \theta, z\}$ coordinates:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_r)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

the continuity equation is: $\frac{1}{r}\frac{\partial (r v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$