2.25 ADVANCED FLUID MECHANICS Fall 2005

QUIZ 2

TUESDAY, November 22, 2005, 7:00-9:00 P.M.

OPEN QUIZ WHEN TOLD AT 7:00 PM

THERE ARE TWO PROBLEMS
OF EQUAL WEIGHT

Please answer each question in SEPARATE books

You may use SEVEN (7) pages of handwritten notes as well as the single page handouts posted on MIT Server plus any tables you feel are necessary

Question 1:

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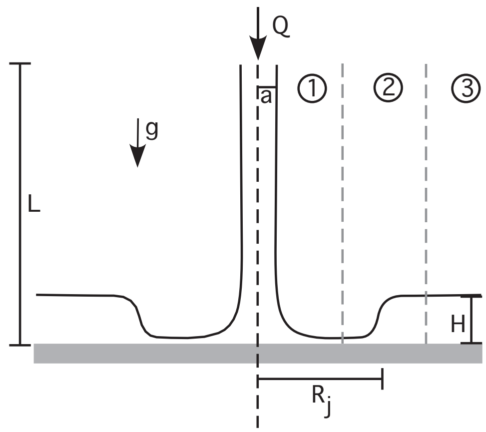


When a jet of fluid impinges on a horizontal plate, the fluid flows radially outward away from the jet in a thin film. At a distance R_j from the jet, the thickness of the film suddenly increases. This phenomenon is known as a hydraulic jump. In general, the flow in the jump region is turbulent.[1] Consider a jet with **volume** flux, Q, and radius, a. The fluid in the jet can be approximated as **inviscid** with density, ρ. The acceleration of gravity is $-g$ in the $\mathbf{e}_z$ direction and the jet inlet is located a distance L above the plate. For radii larger thatn R_j, the fluid free surface is at a constant height, H. You may assume the flow is axially symmetric. You may also assume $a \ll H \ll R_j$.

The hydraulic jump can be divided into three regions as indicated in the figure: (1) the upstream region, $a < r < R_j$, (2) the jump region $r \sim R_j$ and (3) the downstream region $r > R_j$. Follow the steps below to find an expression for the steady state jump radius, R_j.

a.) Write down the important parameters that set the jump radius R_j i.e. $R_j = f(?,?,?...)$. Using dimensional analysis, find a complete set of independent Pi groups for this system. (Remember, Q is **volume** flux NOT mass flux.)

b.) In the downstream region (3), the height of the free surface is a known constant, H (i.e., the height of the free surface is NOT a function of r). Find an expression for the average velocity in the downstream region.

c.) The upstream region (1) consists of a jet impinging on a horizontal plate. Derive an expression for the height of the free surface in this region, $h(r)$, and the average radial velocity. You may simplify your answer by assuming that the

[1]This turbulence may be suppressed in sufficiently viscous fluids but here we will only consider the turbulent case.

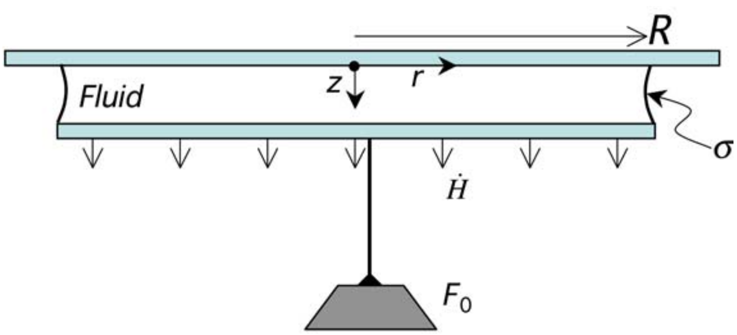
Froude number defined in terms of L, Fr_L, is very large ($Fr_L \gg 1$). (This assumption will make the algebra easier in part (d).)

d.) Now that you have obtained an expression for the height of the free surface and the velocity on either side of the jump, derive an expression for R_j in terms of known parameters.

e.) Convert your answer from part (d) into dimensionless form using your Pi groups from part (a).

2. Adhesive Separation of Two Disks

It is a matter of common experience that it can take a large force to separate two surfaces that are joined by a thin layer of a viscous fluid which acts as an adhesive. Some insects such as aphids are believed to exploit this fact by using thin adhesive-like liquid films on their feet to enable them to walk inverted across the ceiling. In this question we consider the model problem of a cylindrical disk (radius R) separated by a uniform initial thickness H_0 ($\ll R$) with a thin film of a viscous incompressible Newtonian fluid (of viscosity μ and density ρ) in the gap.



The plates are to be separated by a constant force F_0 that is imposed on the lower plate as shown in the figure opposite. We wish to solve for the separation profile $\dot{H}(t)$ as a function of time (where the overdot indicates a time derivative).

(a) The cylindrical fluid film shown has a free surface with a surface tension σ. Use dimensional analysis to provide a dimensionless constraint (in terms of the given variables in the problem) for conditions under which all capillary effects (i.e. interfacial contributions to the force balance) are negligible in the subsequent analysis.

(b) Let us consider the first instant in time (denoted $t = 0^+$) after the constant force is applied to the disk when the sample is cylindrical as shown. Write down the appropriate simplified form of the Navier-Stokes equations (in cylindrical coordinates) together with the appropriate boundary conditions, and any dimensionless criteria which must be attained for this simplification to be valid.

(c) Find the resulting form of the radial velocity profile in terms of the (as yet unknown) radial pressure gradient.

(d) Use conservation of mass to write down a kinematic condition that relates the volumetric inflow through an annulus of radius $r < R$ and the axial displacement rate $\dot{H}(t)$ of the disk. Use this result plus appropriate boundary conditions to find an equation for the pressure profile across the sample. Is the pressure at the center of the disk greater than or less than atmospheric?

(e) Integrate your result to show that the net axial force (arising from viscous flow) acting on the plate is given by (where $H = H(t)$).

$$F_v = -\frac{3\pi}{2}\mu R^4 \frac{\dot{H}}{H^3}$$

(f) Assuming that this expression remains valid for all future times and plate separations, find an expression for the plate separation as a function of time. You may ignore the mass and inertia of the plate and assume the forces are in quasi-steady equilibrium.
$\Rightarrow$ Sketch the profile for $H(t)$ and explain why it shows a finite-time singularity.

(g) The time-dependent solution obtained in (f) must fail at some point. Use dimensional analysis to provide an estimate for a dimensional characteristic time τ for this constant force separation process in terms of the instantaneous position and velocity. Evaluate this expression using your solution for *H(t)* and thus provide a dimensionless constraint (in terms of $H^2/\nu\tau$) for when your solution in (f) is valid.