M-Structures: Programming with State and Nondeterminism

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Lecture 12

Limitations of Functional Programming

- Forces an *obscure coding style* - threading the "state" - for some problems
- Requires too much *storage*
- Cannot express the *parallelism* in some algorithms
- Cannot express *non-deterministic algorithms*
  - histograms
  - graph traversals
- Cannot express *non-determinism inherent in*
  - access to shared resources
  - storage allocator

Language extensions

- I-structures: "write once" variables
  - Multiple writes cause an "inconsistency" and blowup the program. A flavor of logic variables
  - Benign side-effects but equational reasoning is weakened
- M-structures: "synchronized reads and writes".
  - each read "empties" the variable and a write to a "full" variable causes a program blowup
  - simultaneously requires the notion of a "barrier" to control the order of evaluation of some expressions
  - equational reasoning is weakened dramatically
- Monads: a new way of manipulating programs (has become very popular in the last decade)
  - preserves equational reasoning
  - not obvious how to use it for expressing parallelism

Nov 4,6

I-Cell: The Simplest I-Structure

```
data ICell a = ICell {contents :: . a}
```

I-Structure field

Constructor

```
ICell :: a -> ICell a

ICell e              or      ICell {contents = e}
```

or create an empty cell and fill it (a "side-effect")

```
ic = ICell {}
contents ic := e
```

Selector (iFetch)

```
contents ic          or
case ic of
     ICell x -> ... x ...
```

I-Cell: Dynamic Behavior

- Let allocated I-cells be represented by objects o_1, o_2, ...
- Let the states of an I-cell be represented as:

  empty(o) | full(o,v) | error(o)

- When a cell is allocated it is assigned a new object descriptor o and is empty, i.e., empty(o)
- Reading an I-cell
  (x=iFetch(o) ; full(o,v)) $\Rightarrow$ (x=v ; full(o,v))
- Storing into an I-cell
  (iStore(o,v) ; empty(o)) $\Rightarrow$ full(o,v)
  (iStore(o,v) ; full(o,v')) $\Rightarrow$?(error(o); full(o,v'))

Multiple-Store Error

Multiple assignments to an I-cell cause a multiple store error

A program with exposed store error is suppose to blow up!

Program --> T

The Top represents a contradiction

M-Cell: The Simplest M-Structure

```
data MCell a = MCell {contents :: & a}
```

M-Structure field

Constructor

```
MCell :: a -> MCell a
```

`MCell e` *or* `MCell {contents = e}`

or create an empty cell and fill it

```
mc = MCell {}
contents mc := e
```

overloaded notation

Selector (mFetch)

```
contents & mc
```

pattern matching ?

M-Cell: Dynamic Behavior

- Let allocated M-cells be represented by objects o_1, o_2, ...
- Let the states of an M-cell be represented as:

  empty(o) | full(o,v) | error(o)

- When a cell is allocated it is assigned a new object descriptor o and is empty, i.e., empty(o)
- Reading an M-cell
  (x=mFetch(o) ; full(o,v)) ⇒ (x=v ; empty(o))
- Storing into an M-cell
  (mStore(o,v) ; empty(o)) ⇒ full(o,v)
  (mStore(o,v) ; full(o,v')) ⇒?(error(o); full(o,v'))

The Need of Barriers

Suppose we want to replace the contents of M-Cell **mc** by zero.

We need to empty it first to avoid a double store error.

First attempt:

```
let old  = content & mc
    content mc := 0
in ...
```

Correct ?

Second attempt:

M-Cell: Imperative Reads and Writes

Examine: like a read operation

```
contents mc ≡  let v = contents & mc
                   contents mc := v
               in
                   v
```

Replace: like an update operation

```
contents & mc := e   ≡
        v = e
      ( _ = v >>>
        _ = contents & mc >>>
        contents mc := v )
```

M-structures with barriers have the full expressive power of imperative languages *but the language is not sequential!*

M-Arrays

- *Allocate*

  `x = mArray (1,n) []`

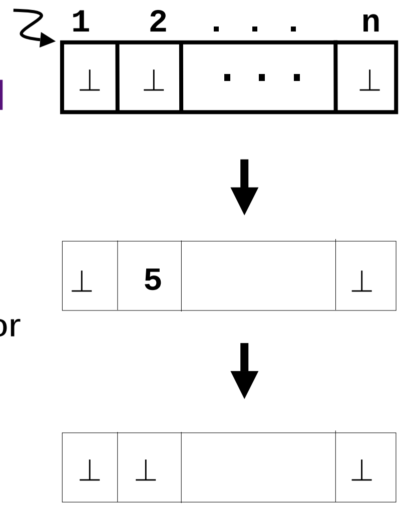
- *Put*

  `x!2 := 5`

  A put operation on
  a full slot is an error

- *Take*

  `x!&2`



Three Examples

- Histograms
- Inserting an element in a list
- Graph traversal

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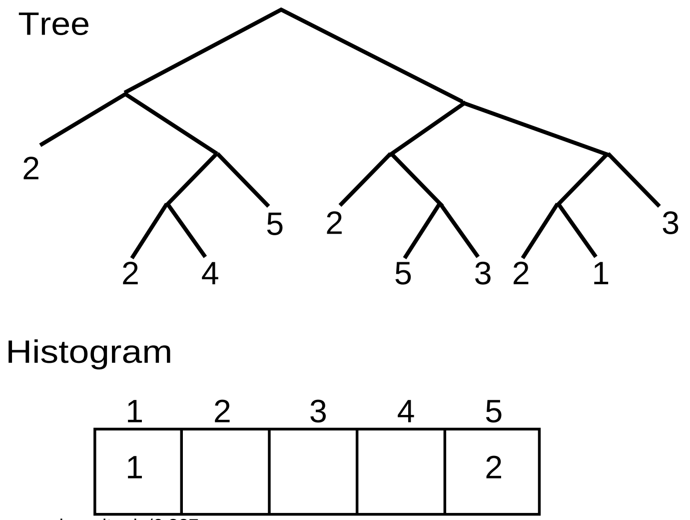
Histogram of Elements in a Tree



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Histogram: A Functional Solution

Thread the histogram array

```
data Tree = Leaf Int | Node Tree Tree
traverse :: Tree ->(ArrayI Int)->(ArrayI Int)
traverse (Leaf i)           hist = incr hist i
traverse (Node ltree rtree) hist =        ?
```

```
incr hist j =
      let inc i = if i == j then (hist!i)+1
                                 else  hist!i
      in  mkArray (bounds hist) inc
```

```
mkHistogram tree =
      let hist = array (1,5) [ 0 | i <- [1..5]]
      in  traverse tree hist
```

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Histogram : Using M-structures

```
mkHistogram tree =
   let  hist = mArray (1,5) [ 0 | i <- [1..5]]
       ( traverse tree hist
         >>>
         hist' = hist )
   in hist'

traverse :: Tree -> (MArrayI Int) -> ()
traverse (Leaf i) hist =                              ?

traverse (Node ltree rtree) hist =                    ?
```

No threading, No copying

+ Natural coding style and more parallelism

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Mutable Lists

Any field in an algebraic type can be specified as an M-structure field by marking it with an "&"

```
data  MList t = MNil
                | MCons {hd::t, tl::&(MList t)}
```

M-structure slot

Allocate

```
x = MCons {hd = 5}
```

Take

```
tl & x
```

Put

```
tl x := v
```

No side-effects while pattern matching

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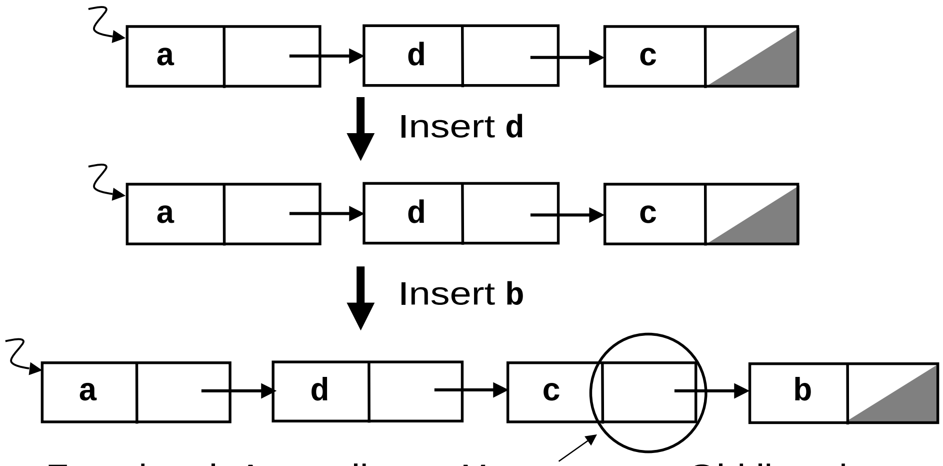
Inserting an element in a list



Functional: A new list whose last element is b

M-structures: Old list whose last element is mutated to point to b

Insert: Functional and Non Functional

Functional solution:

```
insertf  []     x = [x]
insertf (y:ys)  x = if (x==y) then y:ys
                    else y:(insertf ys x)
```

M-structure solution:

```
insertm ys x =
   case ys of
     MNil         -> MCons x MNil
     MCons y ys'  -> if x == y then ys
                     else                ?
```

Subtle Issues

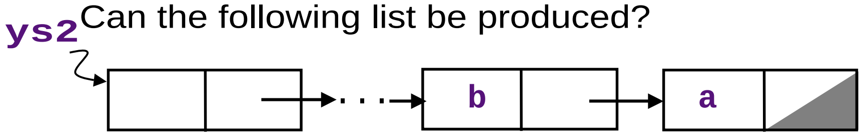
Compare

```
ys1 = insertf ys  a
ys2 = insertf ys1 b
```

```
ys1 = insertm ys  a
ys2 = insertm ys1 b
```

assuming **a** and **b** are not in **ys**.

Can the following list be produced?



Out-of-order Insertion

```
insertm ys x =
  case ys of
     MNil        -> MCons x MNil
     MCons y ys' ->
        if x == y then ys
        else let tl ys := insertm (tl&ys) x
             in  ys

ys1 = insertm ys  a
ys2 = insertm ys1 b
```

ys1 can be returned before the insertion of a is complete but **(tl&ys)** can't be read again before **(tl&ys)** is set

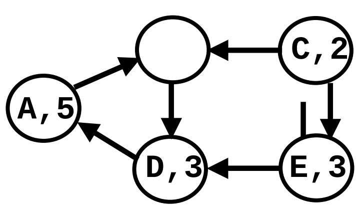
Can you replace **tl&ys** by **ys'**?

Membership and Insertion

insertm' is the same as **insertm** except that it also returns a flag that indicates if a match was found

```
insertm' ys x =
  case ys of
     MNil        -> (False,(MCons x MNil))
     MCons y ys’ ->
        if x == y then (True,ys)
        else let
                (flag,ys’’) = (insertm’ (tl&ys) x)
                 tl ys := ys’’
             in
                (flag, ys)
```


Graph Traversal

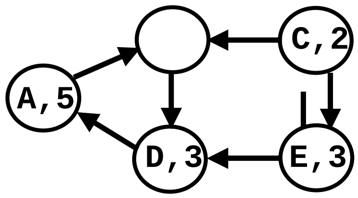



```
data GNode =
      GNode {id  :: Nodeid,
             val :: Int,
             nbrs:: [GNode] }
a = GNode “A” 5 [b]
b = GNode “B” 7 [d]
c = GNode “C” 2 [b]
d = GNode “D” 3 [a]
e = GNode “E” 3 [c,d]
```

Write function **rsum** to sum the nodes reachable from from a given node.

rsum a ==> ?

Graph Traversal: First Attempt




```
data GNode =
      GNode {id  :: Nodeid,
             val :: Int,
             nbrs:: [GNode] }
```

```
rsum (GNode x i nbs) =
                  i + sum (map rsum nbs)
```


Mutable Markings

Keep an updateable boolean flag to record if a node has been visited. Initially the flag is set to false in all nodes.

```
data GNode = GNode {id::Nodeid, val::Int,
                    nbrs::[GNode], flag::&Bool}
```

A procedure to return the current flag value of a node and to simultaneously set it to true

```
marked node = let m = flag & node >>>
                  flag node := True
              in
                     m
```

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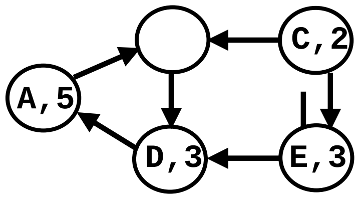
Graph Traversal: Mutable Markings




```
data GNode =
      GNode {id  :: Nodeid,
             val :: Int,
             nbrs:: [GNode]
             flag::&Bool   }
```

```
rsum node =
      if marked node then 0
      else
            (val node)
          + sum (map rsum (nbrs node))
```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Book-Keeping Information

```
data GNode = GNode {id::Nodeid, val::Int,
                    nbrs::[GNode], flag::&Bool}
```

The graph should not be mutated!

Keep the visited flags in a separate data structure - *a notebook* with the following functions

```
mkNotebook :: () -> Notebook
member     :: Notebook -> Nodeid -> Bool
```

Immutable (functional) notebook

```
insert :: Notebook -> Nodeid -> Notebook
```

Mutable notebook: insertion causes a side-effect

```
insert :: Notebook -> Nodeid -> ()
```

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Graph Traversal: *Immutable Notebook*

Thread the notebook and the current sum through the reachable nodes of the graph in any order

```
data GNode =
 GNode {id::Nodeid, val::Int, nbrs::[GNode]}

rsum node =
 let nb = mkNotebook ()        -- a new notebook
     (s,_) = thread (0, nb) node
     thread (s,nb) (GNode x i nbs) =
        if member nb x then (s,nb)
        else let nb' = insert nb x
                 s' = s + i
             in                              ?
 in  s
```

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Graph Traversal: *Mutable Notebook*

```
rsum node =
  let nb = mkNotebook ()       -- a new notebook

      rsum' (GNode x i nbs) =
         if (member nb x) then 0
          else let
                   insert nb x >>>
                   s = i + sum (map rsum' nbs)
               in s
   in rsum' node
```

- *No threading*
- *No copying*

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