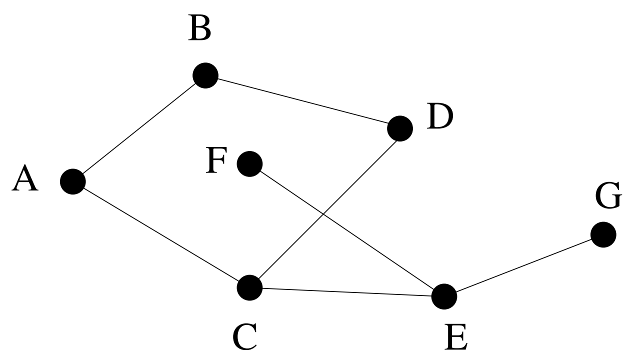
Problems for Recitation 6

1 Graph Basics

Let $G = (V, E)$ be a graph. Here is a picture of a graph.



Recall that the elements of V are called vertices, and those of E are called edges. In this example the vertices are $\{A, B, C, D, E, F, G\}$ and the edges are

$$\{A—B, B—D, C—D, A—C, E—F, C—E, E—G\}.$$

Deleting some vertices or edges from a graph leaves a *subgraph*. Formally, a subgraph of $G = (V, E)$ is a graph $G' = (V', E')$ where V' is a nonempty subset of V and E' is a subset of E. Since a subgraph is itself a graph, the endpoints of every edge in E' must be vertices in V'. For example, $V' = \{A, B, C, D\}$ and $E' = \{A—B, B—D, C—D, A—C\}$ forms a subgraph of G.

In the special case where we only remove edges incident to removed nodes, we say that G' is the *subgraph induced on* V' if $E' = \{(x—y \mid x, y \in V' \text{ and } x—y \in E\}$. In other words, we keep all edges unless they are incident to a node not in V'. For instance, for a new set of vertices $V' = \{A, B, C, D\}$, the induced subgraph G' has the set of edges $E' = \{A—B, B—D, C—D, A—C\}$.

2 Problem 1

An undirected graph G has ***width*** w if the vertices can be arranged in a sequence

$$v_1,\ v_2,\ v_3,\ \ldots,\ v_n$$

such that each vertex v_i is joined by an edge to at most w preceding vertices. (Vertex v_j *precedes* v_i if $j < i$.) Use induction to prove that every graph with width at most w is $(w+1)$-colorable.

(Recall that a graph is k-*colorable* iff every vertex can be assigned one of k colors so that adjacent vertices get different colors.)

3 Problem 2

A **planar graph** is a graph that can be drawn without any edges crossing.

1. First, show that any subgraph of a planar graph is planar.

2. Also, any planar graph has a node of degree at most 5. Now, prove by induction that any graph can be colored in at most 6 colors.

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