Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.685 Electric Machines

Problem Set 4 Solutions October 10, 2005

Problem 1: The problem statement should have led you through this, so we leave some of the statement here:

1. To start, note that this machine will have a stability limit for operation at low field excitation (corresponding to high absorbed reactive power). For a round rotor machine this limit is reached at a torque angle of 90°, but this machine has saliency so you must determine the value of angle for which stability is reached. Compute and plot the angle and corresponding value of field current at the stability threshold for this machine, against real power. The stability limit is reached when the derivative of torque with respect to angle is zero. Since torque is proportional to real power, you can use the derivative of power with angle.

   Real power output for a generator is:

$$P = 3\frac{VE_{af}}{X_d}\sin\delta + \frac{3}{2}V^2\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta$$

   The derivative of power with angle is then simply:

$$\frac{dP}{d\delta} = 3\frac{VE_{af}}{X_d}\cos\delta + 3V^2\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\cos 2\delta$$

   At the stability limit, $\frac{dP}{d\delta} = 0$, and this may be solved for internal voltage:

$$E_{af} = -V\left(\frac{X_d}{X_q} - 1\right)\frac{\cos 2\delta}{\cos\delta}$$

   Using this shorthand:

$$P_0 = 3V^2\left(\frac{1}{X_q} - \frac{1}{X_d}\right)$$

   we have this nonlinear expression to solve:

$$\frac{P}{P_0}\cos\delta - \frac{1}{2}\sin 2\delta + \cos 2\delta \sin\delta = 0$$

   Now, this looks awful but in fact is quite easily solved by most mathematical assistants. MATLAB, for example, has a routine called 'fmins' which makes quick work of it. Once δ is found, E_{af} may be determined and the operating point is easily determined. The values for angle δ and field current $I_f = E_{af}/Xm$ are plotted in Figure 1

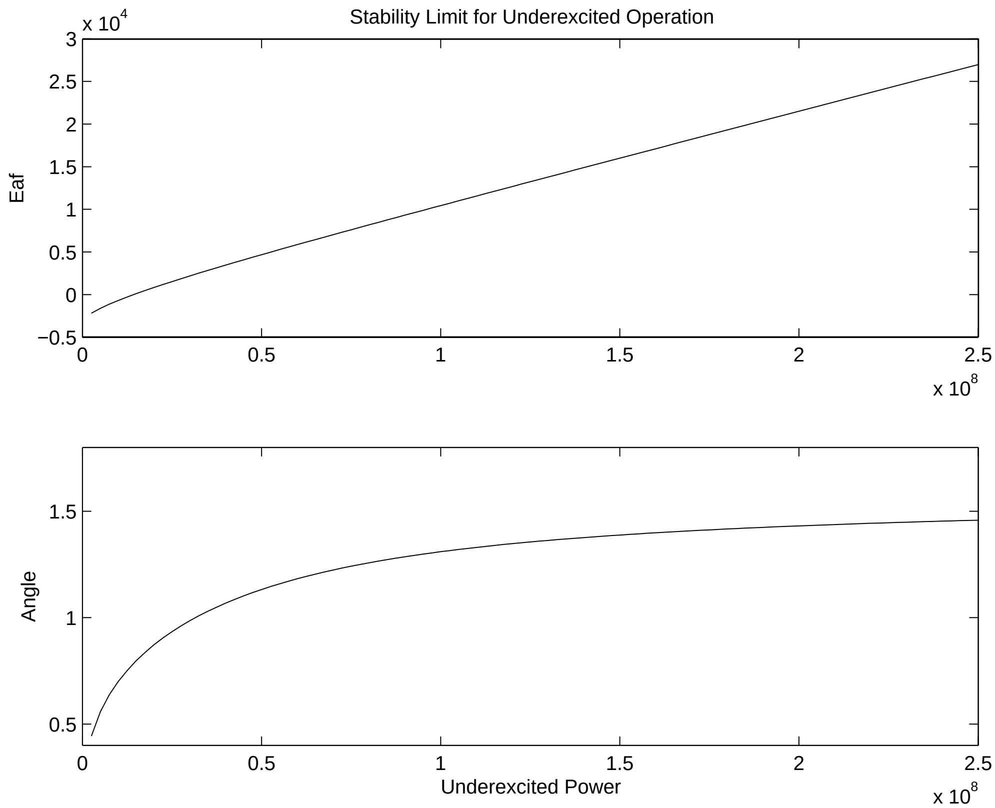

Figure 1: Angle and Field Current at Stability Limit

2. Since the capability curve is to be plotted as reactive power (Q) vs real power (P), you must determine the value of Q at the stability limit. Actually, the underexcited reactive power limit may be either statitility or armature (current) capacity. So determine the underexcited limit Q as a function of real power P.

   We use, of course:

$$Q = 3\frac{VE_{af}}{X_d}\cos\delta + \frac{3}{2}V^2\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\cos 2\delta - \frac{3}{2}V^2\left(\frac{1}{X_q} + \frac{1}{X_d}\right)$$

   Now, if it possible that this stability limited value of Q is outside the armature capacity. That is easily checked:

$$|Q| = \sqrt{VA^2 - P^2}$$

3. There is also a limit for over-excited operation. That limit might be field current and it might be armature current. To establish the field current limit, assume that this machine can reach the armature current limit for power factor of 0.8 and above, but for power factors below that the machine is field current limited. Find the torque angle and corresponding field current limits for over-excited operation at the defining power factor.

It is straightforward to calculate the angle δ at the specified value of power factor: using the power factor angle:

$$\delta = atan \frac{X_q I \cos\psi}{V + X_q I \sin\psi}$$

where I is the armature limited stator current:

$$I = \frac{VA}{3V}$$

4. For torque angles less than this power-factor determined condition, real and reactive power are simply determined by torque angle and internal voltage E_{af} which is fixed at its limit. These are a simple plug-in to the existing formulae.
5. For values of real power greater than this power factor determined situation real and reactive power are on the armature current (heating) limit and so are easy to compute.
6. These were all added together (see attached script) and the result is the rather odd looking Figure 2

Problem 2: Figure 3 shows, in very schematic form, the situation. Pictured is only the air-gap, not for clairity but to simplify what has to be drawn. Each slot has two coil halves, each carrying 10,000 Ampere-Turns. The slots are 90 physical degrees or 180 electrical degrees apart.Flux density across the air-gap must then be as shown in Figure 4, where the amplitude is:

$$B_a = \mu_0 \frac{NI}{2g} = 1.2566 \times 10^{-6} \times \frac{20000\text{A-T}}{2 \times .05} \approx .2512\text{T}$$

The Fourier Series for this is:

$$B_r = \sum_{\text{n odd}} \frac{4}{n\pi} \frac{\mu_0 NI}{2pg} \sin np\theta$$

It is straightforward to compute this Fourier Series (see the attached script), and the resulting picture is plotted in Figure 5. In this figure we have plotted the sum of Fourier harmonics up to ninth. The 'exact' solution is also plotted as a dotted line.

Now, were we to put these coils in adjacent slots as is shown in Figure 6, the slot spacing is $\gamma = \frac{2\pi}{12} = 30°$. The *step* in flux density is the same as in the single coil case (across each of the two coils). The peak flux density is twice that of the previous case, as is shown in Figure 7

If we short-pitch this winding by one slot, we wind up putting NI ampere-turns in the outer of three slots and $2NI$ in the inner (central) slot, as shown in Figure 8. The flux density from this arrangement has the same peak value but the steps are different, as shown in Figure 9

Now, it is possible to use the method of pitch and breadth factors to reproduce these flux density patterns. See that, for a short-pitched winding with pitch angle α, the pitch factor is defined by:

$$k_p = \frac{2}{\pi} \int_{\frac{\pi}{2}-\frac{\alpha}{2}}^{\frac{\pi}{2}+\frac{\alpha}{2}} \sin n\theta d\theta = \frac{4}{n\pi} \sin\left(n\frac{\pi}{2}\right) \sin\left(n\frac{\alpha}{2}\right)$$

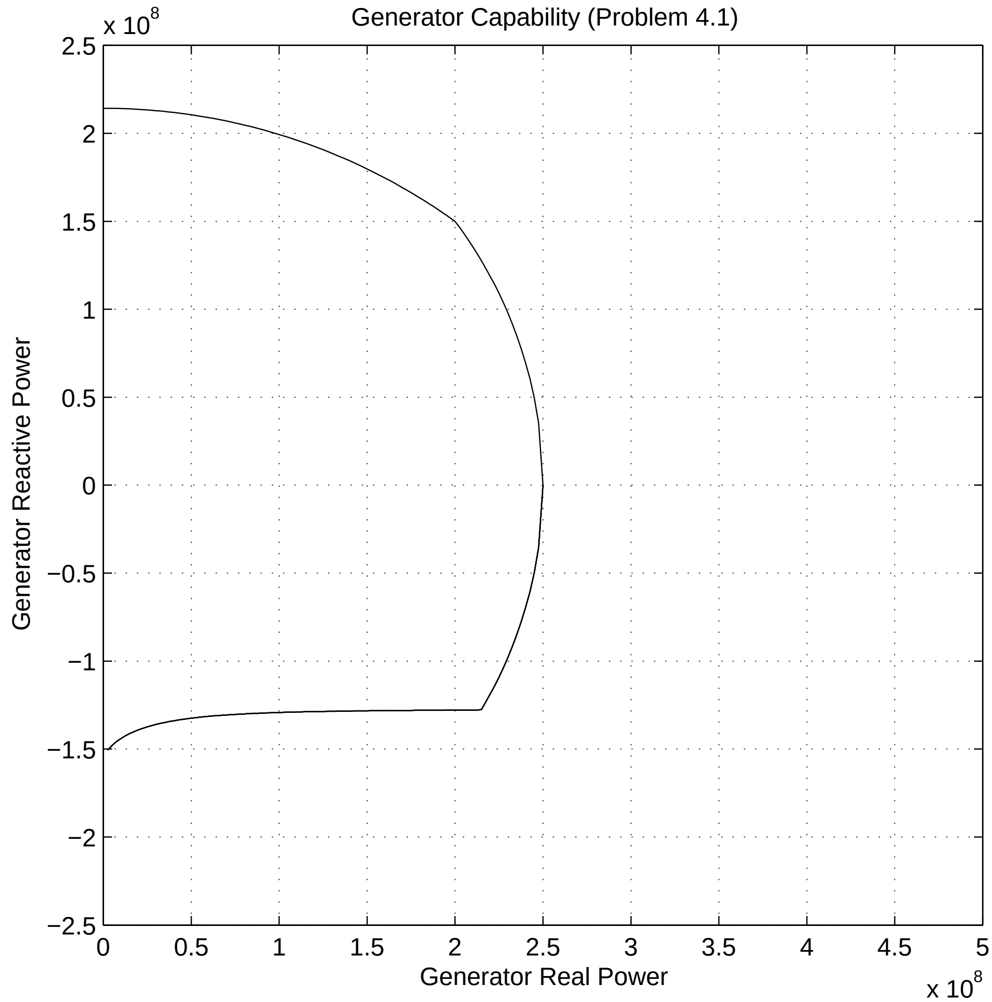

Figure 2: Capability Chart of Problem 1

and the breadth factor is, from the notes:

$$k_b = \frac{\sin\left(mn\frac{\gamma}{2}\right)}{m\sin\left(n\frac{\gamma}{2}\right)}$$

For the full-pitch case, use $\gamma = \frac{\pi}{6} = 30^\circ$, and the pitch factor is unity. For the short-pitch case, $\alpha = \pi - \frac{\pi}{6} = 150^\circ$. The two cases are computed and plotted in Figure 10

Finally, to find winding inductance we use the expression for the space fundamental part of inductance:

$$L_1 = \frac{3}{2}\frac{4}{\pi}\frac{\mu_0 N_a^2 RLk_w^2}{p^2 g}$$

Here the number of turns is $N_a = 2pmN = 8$.This assumes each of the four poles is surrounded by two turns.

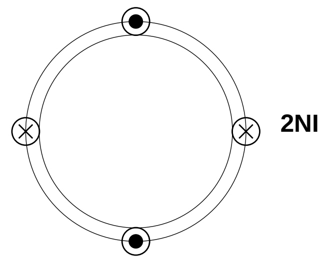

Figure 3: Full-Pitch Coil Cartoon

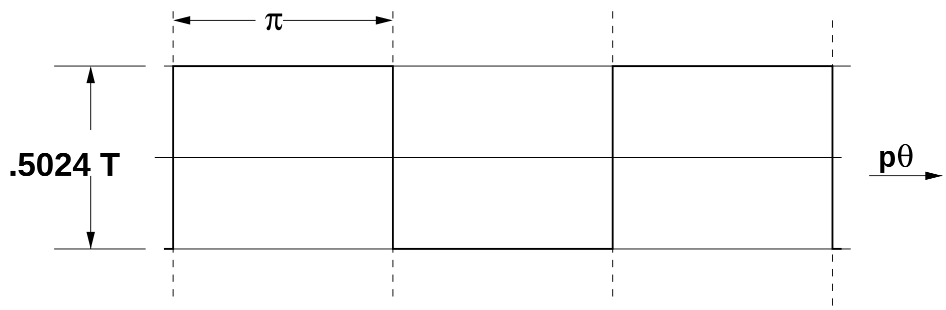

Figure 4: Full-Pitch Coil Flux Density

In the problem statement I neglected to say for which winding to compute inductance. The only difference is the winding factor, which for the full-pitched winding is about .9659, while for the short pitched winding it is about .9330. Working out the inductance (actually the script does this), we find L_1 to be about 2.15 mHy for the full-pitch case and about 2.01 mHy for the short pitch case.

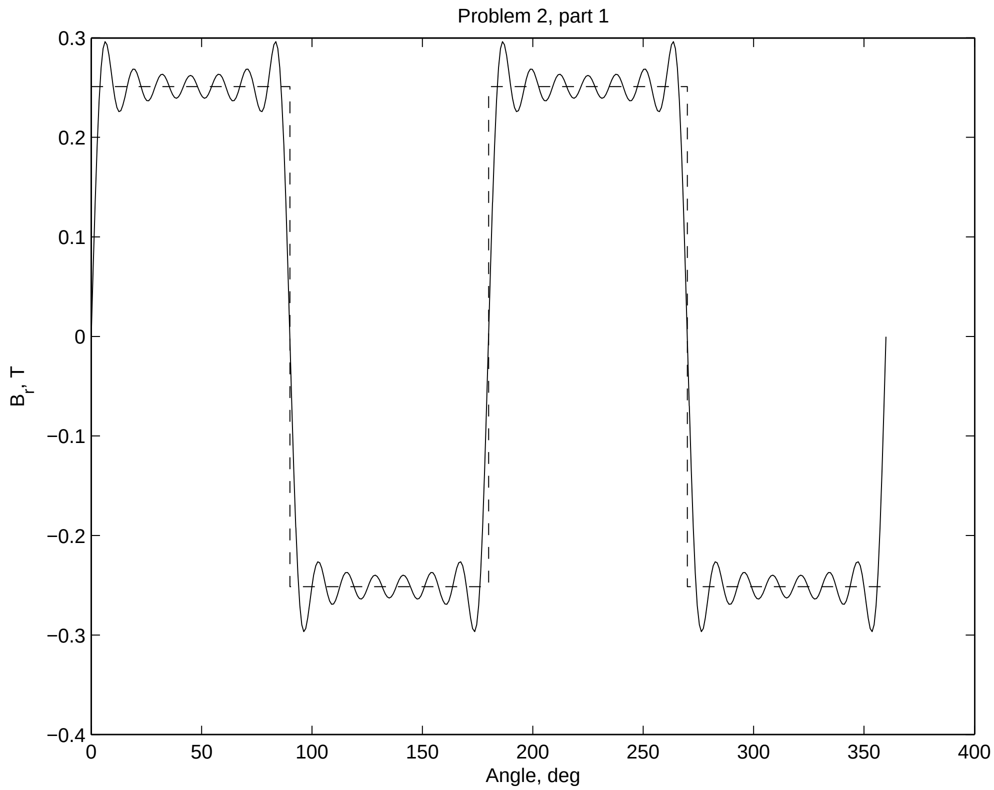

Figure 5: Full-Pitch Coil Flux Density

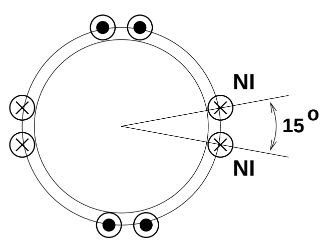

Figure 6: m=2, Full-Pitch Coil Arrangement

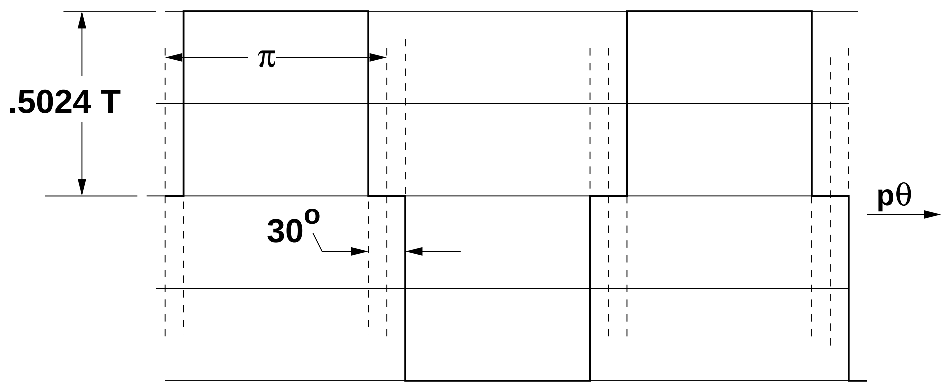

Figure 7: Flux Density from m=2, Full Pitch

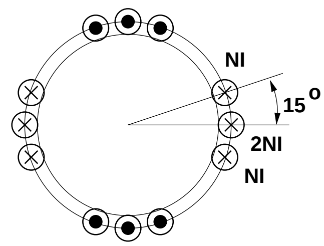

Figure 8: m=2, Short-Pitch Coil Arrangement

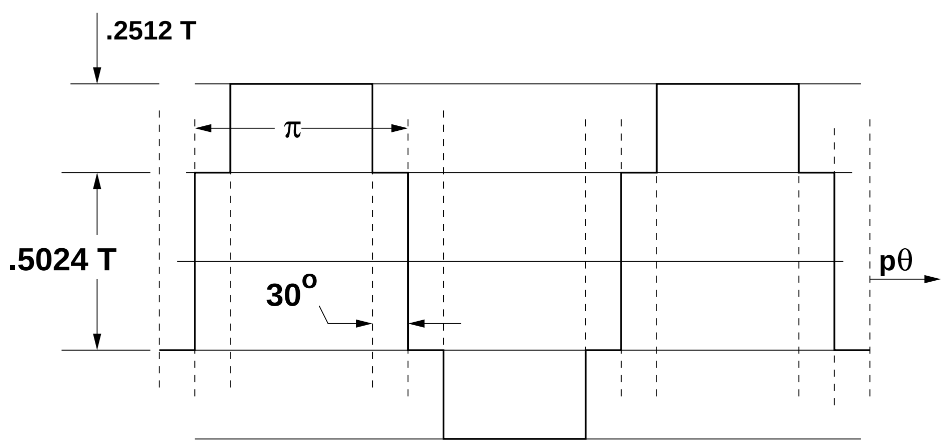

Figure 9: Flux Density from m=2, Short Pitch

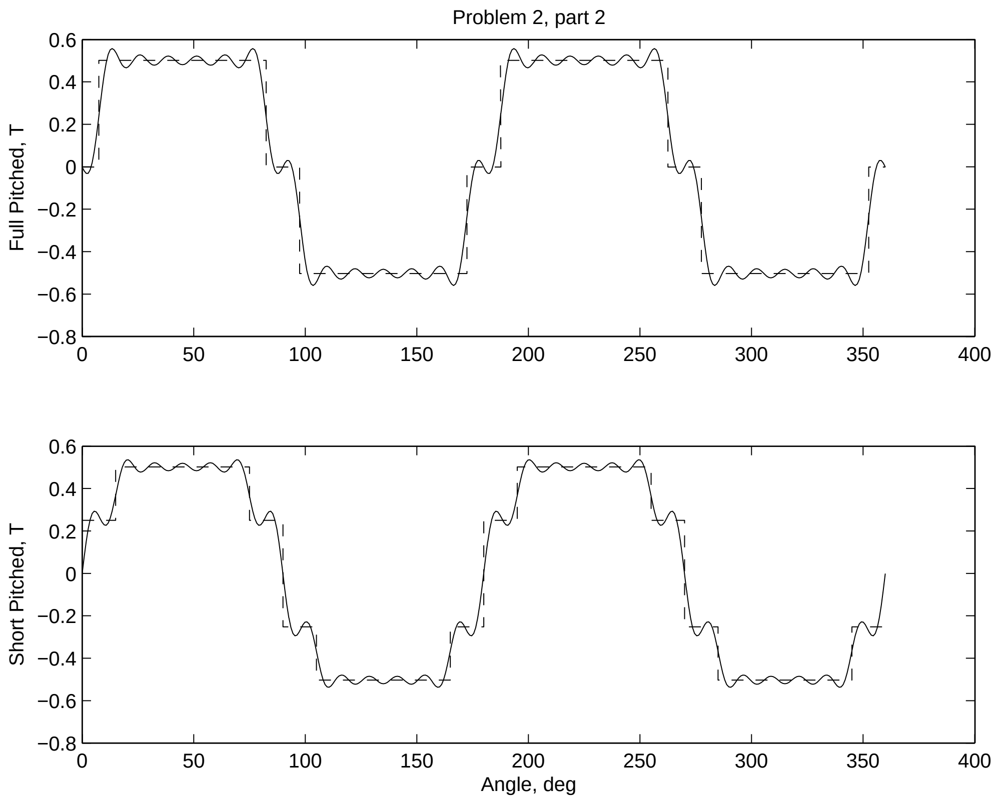

Figure 10: Full-Pitch Coil Flux Density

```
% 6.685 Problem Set 4, Problem 1
% First, line up parameters
VA=250e6;                   % Machine Rating
V = 24000/sqrt(3);          % Phase voltage, RMS
om = 2*pi*60;               % frequency
Xd = .012*om;               % direct axis reactance
Xq = .0098*om;              % quadrature axis reactance
Xm = .052*om;               % mutual reactance
pfr = 0.8;                  % can operate to this level of over-excited power fac

% We are going to trace out the capability chart for this machine
% First, get over-excited limit
psi = acos(pfr);            % limiting power factor angle
Ial = VA/(3*V);             % armature current limit
dl = atan(Xq*Ial*cos(psi)/(V+Xq*Ial*sin(psi)));
E_1 = sqrt((V+Xq*Ial*sin(psi))^2+(Xq*Ial*cos(psi))^2);
I_d = Ial*sin(dl+psi);
Eafl = E_1 + I_d*(Xd-Xq);

delt_1 = 0:.01:1 .* dl;       % range of angles
P_1 = (3*V*Eafl/Xd) .* sin(delt_1) + 1.5*V^2*(1/Xq-1/Xd) .* sin(2 .* delt_1);
Q_1 = (3*V*Eafl/Xd) .* cos(delt_1) + 1.5*V^2*(1/Xq-1/Xd) .* cos(2 .* delt_1) -

P_2 = (pfr:.01:1) .* VA;       % rest of over-excited range
Q_2 = sqrt(VA^2 - P_2 .^2);

% Now for under-excited: Find Stability Limit
% This will be angle delta as a function of real power P
warning off MATLAB:fzero:UndeterminedSyntax   % to suppress a whole lot of wierd
P_0 = 3*V^2*(1/Xq-1/Xd);        % convenient shorthand
P_3 = (1:-.01:.01) .* VA;         % establish a range (note we are going down!
P_3c = zeros(size(P_3));        % gonna check ourselves here
Q_3 = zeros(size(P_3));         % space for Q
E_af = zeros(size(P_3));
ds = zeros(size(P_3));

for i = 1:length(P_3)
    Pr = P_3(i)/P_0;          % here is how we use the notation
    d = fzero('ef', [0 pi/2], [], Pr);  % this gives angle at stability limit
    Eaf = -V*(Xd/Xq-1)*cos(2*d)/cos(d); % and corresponding internal voltage
    E_af(i) = Eaf;
    ds(i) = d;
    P_3c(i) = 3*V*Eaf*sin(d)/Xd + 1.5*V^2*(1/Xq-1/Xd)*sin(2*d);
    % need to check on stability limited or curent limited Q
    Qs = (3*V*Eaf/Xd) * cos(d) + 1.5*V^2*(1/Xq-1/Xd) * cos(2*d) - 1.5*V^2*(1/Xq+1
```

```
    Qc = sqrt(VA^2-P_3(i)^2);
    Q_3(i) = max(Qs, -Qc);
end
dpdd = (3*V/Xd) .* E_af .* cos(ds) +3*V^2*(1/Xq-1/Xd) .* cos(2 .* ds);

% We can now tote these things up
P = [P_1 P_2 P_3];
Q = [Q_1 Q_2 Q_3];

figure(1)
plot(P, Q, P_3c, Q_3)  % should get only one curve
title('Generator Capability (Problem 4.1)')
ylabel('Generator Reactive Power')
xlabel('Generator Real Power')
axis([0 5e8 -2.5e8 2.5e8])
axis square
grid on

figure(2)
subplot 211
plot(P_3, E_af)
title('Stability Limit for Underexcited Operation')
ylabel('Eaf')
subplot 212
plot(P_3, ds)
ylabel('Angle')
xlabel('Underexcited Power')
```

```
% 6.685 Problem Set 4, Problem 2
% Field from a simple winding
% Parameters:
muzero=pi*4e-7;
L = 3;
R = 1;
g = .05;
p=2;
N = 1;
I = 10000;

% Here is the basic field amplitude: Two half turns in a slot
Ba = 2*muzero*N*I/(2*g);

% now generate a Fourier Series:
n = 1:2:13;
th = 0:pi/100:4*pi;
Bh = (4*Ba/pi) ./ n;
B = zeros(size(th));
angle = (90/pi) .* th;

for i = 1:length(n)
    B = B + Bh(i) .* sin(n(i) .* th);
end

% and here is the 'exact' solution:
Be = [Ba Ba -Ba -Ba Ba Ba -Ba -Ba];
Ae = [0 90 90 180 180 270 270 360];
figure(1)
plot(angle, B, Ae, Be, '--')
title('Problem 2, part 1')
ylabel('B_r, T')
xlabel('Angle, deg')

% Now the multiple slot sollution
m = 2;
gamma = pi/6;
alfa = pi-gamma;         % short pitched by one slot...

kp = sin(n .* pi/2) .* sin(n .* alfa/2);
kb = sin((m*gamma/2) .* n) ./ (m .* sin((gamma/2) .* n));

% First, full pitched, then shorted by one slot
kw = kp .* kb;
Bh = 2*(4*Ba/pi) .* kb ./ n;
```

```
Bs = 2*(4*Ba/pi) .* kw ./ n;
B_f = zeros(size(th));
B_s = zeros(size(th));

for i = 1:length(n)
    B_f = B_f + Bh(i) .* sin(n(i) .* th);
    B_s = B_s + Bs(i) .* sin(n(i) .* th);
end

% and the 'exact' solutions
Bfe = [0 0 2*Ba 2*Ba 0 0 -2*Ba -2*Ba 0 0 2*Ba 2*Ba 0 0 -2*Ba
Afe = [0 7.5 7.5 82.5 82.5 97.5 97.5 172.5 172.5 187.7 187.5 262.5 262.5 2
Bse = [Ba Ba 2*Ba 2*Ba Ba Ba -Ba -Ba -2*Ba -2*Ba -Ba -Ba Ba Ba 2*Ba 2*Ba
Ase = [0 15 15 75 75 90 90 105 105 165 165 180 180 195
figure(2)
subplot 211
plot(angle, B_f, Afe, Bfe, '--')
title('Problem 2, part 2')
ylabel('Full Pitched, T')
subplot 212
plot(angle, B_s, Ase, Bse, '--')
ylabel('Short Pitched, T')
xlabel('Angle, deg')

Na = m*p*N;

L_1 = (3/2)*(4/pi)*(muzero*Na^2*kw(1)^2*R*L)/(p^2*g)
```