2.830J / 6.780J / ESD.63J Control of Manufacturing Processes (SMA 6303)
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Massachusetts Institute of Technology
Department of Mechanical Engineering
2.830/6.780J Control of Manufacturing Processes
Quiz 2 Solution

Problem 1 (45%)

a)

I	A	B	C	AB	AC	BC	ABC
1	-1	-1	1	1	-1	-1	1
1	-1	1	-1	-1	1	-1	1
1	1	-1	-1	-1	-1	1	1
1	1	1	1	1	1	1	1

b) A=BC, B=AC, C=AB, I=ABC

c) Assign AC to the column currently representing C. This will have the effect of confounding B with C.

d)

Run #	A	B	C
1	-1	-1	-1
2	-1	1	1
3	1	-1	-1
4	1	1	1

e) One additional row is needed. This row must distinguish C from B, so their levels must be different, for instance:

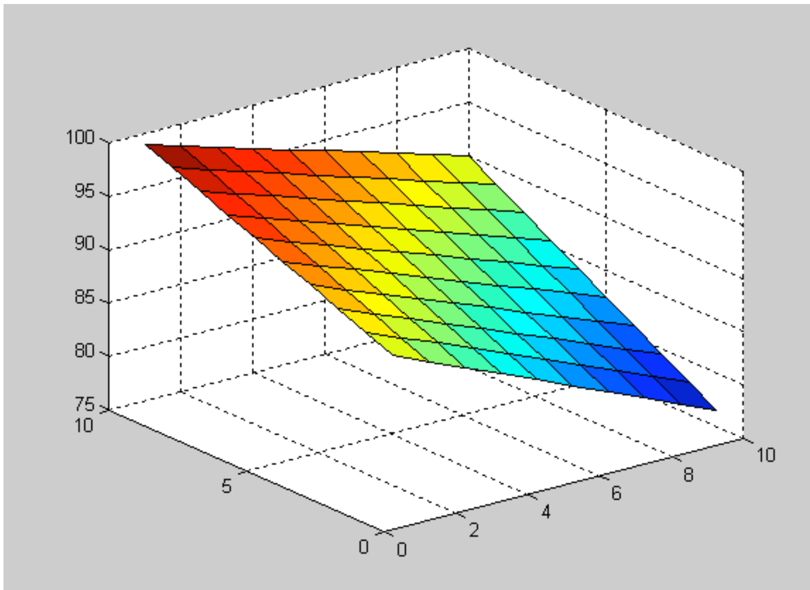
A	B	C	AC
1	1	-1	1

f) The above design is not balanced. The levels of the three inputs could be adjusted so the coded columns will add to zero and the design will be balanced, for example:

Run #	A	B	C
1	-1.5	-1.5	-1
2	-1.5	1	1.5
3	1	-1.5	-1
4	1	1	1.5
5	1	1	-1

Problem 2 (20%)

a) A 2^2 experiment would be needed to distinguish the 2-factor interaction
b) 90 (the β_0 term is the grand mean)
c)



d) Since the mean is specified and there is no model of the variance, the optimum would be that setting that give the target output, and then either maximizes rate or minimizes cost (or some combination of there. For this we set y=85 and solve:

$$85 = 90 + 10x_1 - 15x_2 + 5x_1x_2$$

$$15x_2 - 5 = 10x_1 + 5x_1x_2$$

$$15x_2 - 5 = x_1(10 + 5x_2)$$

$$x_1 = \frac{15x_2 - 5}{10 + 5x_2}$$

x_1 and x_2 must satisfy the above equation to produce y=85. Since maximizing feedrate maximizes production rate, the optimum would be to use the maximum feedrate setting , assuming no too wear change means no effect on cost.

e) The variance is minimized when the gradient is zero. Substitute the above relation into the variance model, take the derivative and solve for x_2, then use the above relation to find x_1.

$$\frac{\partial \sigma^2}{\partial x_2} = 0 = \frac{7}{(x_2 + 2)^2} - 1.12$$

$$\therefore \{x_1, x_2\} = \{0.2, 0.5\}$$

This ignores the optimization of rate, so now their could be a rate – quality tradeoff to find the final value for the feedrate.

Problem 3 (35%)

a) From the mean values and variances we can back calculate the correct sums of squares and mean squares. For example, the SS $_{within}$ of any row is simply s^2 * (n-1). The $SS_{between+}$ is then found from summing the individual deviations of each row average from the grand mean. From this we can get:

ANOVA				
	SS	dof	MS	F
Between	3.13	(a-1) = 3	3.13/3 = 1.04	F =1.04/0.25=4.27
Within	13.7	a(n-1)=56	13.7/56=0.25	
Total	5.18	59		

b) F_{crit}=F(0.05,3,56) The table in the textbook does not have a row for dof=60, so we use dof=60. F_{crit}=3.34. <2.84, so the between variation is significant at the 95% confidence level.

c) If we repeat the above for just the first and last rows we get:

	SS	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	0.19	1	0.19	1.18	0.29	4.20
Within Groups	4.57	28	0.16			
Total	4.76	29				

So now the F test says that there was no significant effect of temperature, so he appears to be wrong, and the extra tests were needed.

d) Clearly the process in not in a state of statistical control, and it is trending down. The other 3 runs are even worse, so all the results are nullified!