

Code No: 132AB

R16

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech I Year II Semester Examinations, August/September - 2017

MATHEMATICS-II

(Common to EEE, ECE, CSE, EIE, IT)

Time: 3 hours

Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

- 1.a) Find $L^{-1}\left(\frac{1}{s^2 - 2s + 5}\right)$ [2]
- b) Let $L\{f(t)\} = \bar{f}(s)$, Prove that $L\{f(at)\} = \frac{1}{a} \bar{f}\left(\frac{s}{a}\right)$. [3]
- c) Define Gamma function. [2]
- d) Evaluate $\int_0^{\frac{\pi}{2}} \sin^3 x \cos^5 x dx$ using Beta and Gamma function. [3]
- e) Evaluate $\int_0^1 \int_0^3 \int_0^4 (x + y + z) dx dy dz$ [2]
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- f) Find the area of the circle using double integral. [3]
- g) Give short note on gradient of a scalar point function. [2]
- h) Find the scalar potential function of an irrotational vector $\bar{f} = x\bar{i} + y\bar{j} + z\bar{k}$ [3]
- i) Evaluate $\oint_C x dx + y dy + z dz$ where C is a circle $x^2 + y^2 = 1$ in xy -plan. [2]
- j) Apply Gauss divergence theorem to evaluate $\iint x dy dz + y dz dx + z dx dy$ over the surface of the sphere of radius 'a' units. [3]

PART-B

(50 Marks)

- 2.a) Find Laplace transform of $\frac{1 - e^t}{t}$.
- b) Find the inverse Laplace transform of $\frac{1}{(s^2 + 1)^2}$. [5+5]
- OR
- 3.a) Solve $y'' + 4y = 0$, $y(0) = 1$, $y'(0) = 6$ using Laplace transform.
- b) Solve the integral equation $f(t) = at + \int_0^t f(u) \sin(t-u) du$, $t > 0$. [5+5]

4.a) Show that $\int_0^{\infty} x^n e^{-ax^2} dx = \frac{1}{2a^{n+1}} \Gamma\left(\frac{n+1}{2}\right)$, ($n > -1$).

b) Evaluate $\int_0^1 x^4 \left(\log \frac{1}{x}\right)^3 dx$. [5+5]

OR

5.a) Prove that $\beta(m, n) = \frac{\sqrt{\pi} \Gamma(n)}{2^{2n-1} \Gamma(n + \frac{1}{2})}$

b) Evaluate $\int_0^1 \frac{x^3}{\sqrt{1-x}} dx$. [5+5]

6. Evaluate $\iint_R xy \, dx \, dy$ where R is the region bounded by x-axis and $x = 2a$ and the curve $x^2 = 4ay$. [10]

OR

7.a) Using triple integral, find the volume of the sphere whose radius is 'a' units.

b) Evaluate $\int_0^{\pi} \int_0^{\sin \theta} r \, dr \, d\theta$. [5+5]

8.a) Find $\text{Curl } \vec{F}$ at the point (1,2,3), given that $\vec{F} = \text{grad}(x^3y + y^3z + z^3x - x^2y^2z^2)$.

b) Find the Directional derivative of $\phi = x^4 + y^4 + z^4$ at the point A(1,-2,1) in the direction of AB where B is (2, 6, -1). [5+5]

OR

9.a) If the vector field $\vec{F} = (2xyz^2)\vec{i} + (x^2z^2 + z \cos yz)\vec{j} + (2x^2yz + y \cos yz)\vec{k}$ is conservative then find its scalar potential function.

b) Prove that $\text{div}(r^n \vec{r}) = (n+3)r^n$ Where $r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$ [5+5]

10.a) Use Divergence theorem to evaluate $\int_S \vec{F} \cdot \vec{n} \, ds$ where $F=3xi+3yj+3zk$ and surface of sphere is $x^2+y^2+z^2 = a^2$.

b) Calculate the work done by a force $F=3xy \vec{i} - y^2 \vec{j}$ in moving a particle in xy- plane from (0, 1) to (1, 2) along the parabola $y = x^2$. [5+5]

OR

11. Verify Stokes theorem for $F=(x^2+y^2)\vec{i}-2xy\vec{j}$ taken around a rectangle bounded by the lines $x = a$, $x = -a$, $y = 0$ and $y = b$. [10]