Code No: R09221801 **R09** **Set No. 2**

II B.Tech II Semester Examinations,April/May 2012
MATHEMATICS - III
Metallurgy And Material Technology

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
All Questions carry equal marks

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1. Evaluate $\int_0^{1+i}(x-y+ix^2)dx$
   (a) along the straight line from z = 0 to z = 1+i
   (b) along the real axis from z = 0 to z = 1 and then along a line parallel to imaginary axis from z = 1 to z = 1+i [15]

2. Find the Taylor's and Laurent's series which represents the function $\frac{1}{(1+z^2)(z+2)}$ when
   (a) $1 \leq |z| \leq 2$
   (b) $|z| \leq 1$
   (c) $|z| \geq 2$ [15]

3. (a) Prove that z^n (n is a + ve integer) is analytic and hence find its derivative.
   (b) Find the regular function whose imaginary part is $e^{-x}(x\cos y + y\sin y)$ [15]

4. (a) Derive Legender's function of second kind.
   (b) Prove that $\int_{-1}^{1} xP_n(x)P_{n-1}(x) = \frac{2n}{4n^2-1}$ [15]

5. (a) Find the Residues of f (z) = $\frac{z^3}{(z-1)^4(z-2)(z-3)}$.
   (b) Evaluate by the method of complex variable the integral $\int_{-\infty}^{\infty} \frac{x^2}{(1+x^2)^3}\,dx$. [8+7]

6. (a) Find the map of the circle $|z| = c$ under the transformation $w = z - 2 + 4i$
   (b) Show that the transformations $w = \frac{z-i}{z+i}$ transform $|w| \leq 1$ into upper half plane I(z)>0. [7+8]

7. (a) Show that a simple graph with n vertices and k components can have $\frac{(n-k)(n-k+1)}{2}$ edges.
   (b) Find the incidence matrix of the following graph (Figure 1). [15]

8. Solve the equation $4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$ by power series method. [15]

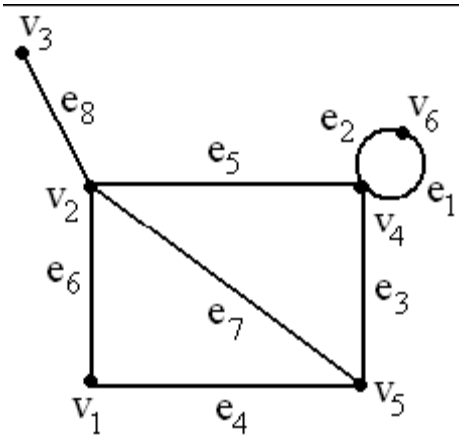

Figure 1

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Code No: R09221801 **R09** **Set No. 4**

II B.Tech II Semester Examinations,April/May 2012
MATHEMATICS - III
Metallurgy And Material Technology

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
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1. (a) Evaluate $\int_C \frac{z^2+2}{(z-1)(z-2)^2}dz$ where C: $\left|z-\frac{1}{2}\right| = \frac{5}{2}$.
   (b) Evaluate $\int_c \frac{dz}{z^8(z+4}$ where C: $|z| = 2$. [15]

2. (a) Show that the function f(z) = $\frac{\bar{z}}{z}$ is not continuous at z = 0.
   (b) Find all values of k, such that f(z) = e^x (cosky + sin ky) is analytic. [15]

3. (a) Prove that $(1 - x^2)P_n'(x) = (n+1)\{xP_n(x) - P_{n+1}(x)\}$
   (b) Express the $1+x-x^2$ interms of legendre polynomials . [15]

4. (a) Find the incidence matrix to represent the following graph (Figure 2).
   (b) Draw the graph of the expression $((x + y) \uparrow 2) + ((x - 4)/3)$ and write its postfix notation by Traversing the tree in post order. [8+7]

5. (a) Prove that $\int J_3(x)dx = -J_2(x) - \frac{2}{x}J_1(x)$
   (b) Show that $J_n(-x) = (-1)^n J_n(x)$: n is a +ve or -ve integer. [15]

6. Determine the poles and their orders for the function f(z) = $(z+1)\sin\frac{1}{z-2}$. What kind of singularity it is? [15]

7. (a) Evaluate $\int_0^\infty \frac{Cos\,2ax - Cos2bx}{x^2}\,dx$ $(a \geq b \geq 0)$ by using complex integration.
   (b) Find the residues of f (z) = $\frac{ze^z}{(z-a)^3}$ at its pole [8+7]

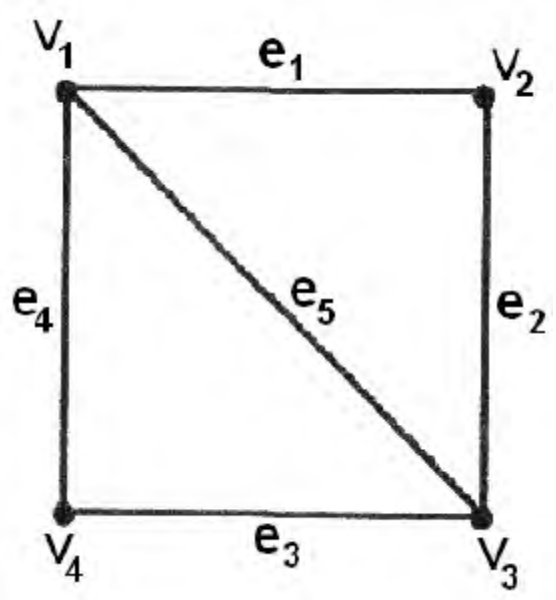

Figure 2:

8. Find the bilinear transformation which maps $(1+i), -i, (2-i)$ of the z-plane in to the points 0,1, i respectively of the w-plane. Find the fixed and critical points of this transformation. [15]

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Code No: R09221801 **R09** **Set No. 1**

II B.Tech II Semester Examinations,April/May 2012
MATHEMATICS - III
Metallurgy And Material Technology

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
All Questions carry equal marks

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1. Integrate f(z) = x^2 + ixy from A(1,1) to B(2,8) along
   (a) the straight line AB
   (b) The curve C:x = t, y = t^3 [15]
2. (a) Find the residue of f(z) = $\frac{z^3}{(z-1)^4(z-2)(z-3)}$ at z = 1,by finding the Laurent's series expansion.
   (b) Show that $\int_0^\infty \frac{\sin x\, dx}{x} = \frac{\pi}{2}$ [7+8]
3. (a) Evaluate $\int_{-1}^{1} x^4(1-x^2)^{-\frac{1}{2}} P_2(x)dx$.
   (b) Prove that $\int_{-1}^{1} x^2 P_{n-1}(x)P_{n+1}(x)dx = 0$ [15]
4. (a) Find the in- degree and out- degree of each vertex of the following graph (Figure 3).

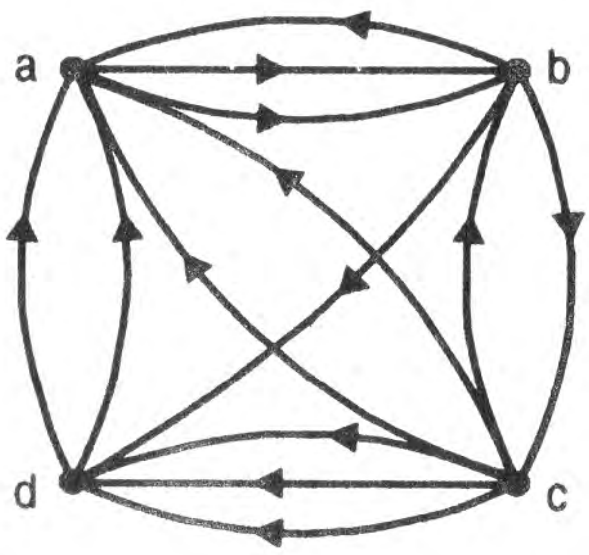

Figure 3:

   (b) Show that every complete graph is regular. [8+7]

5. Find the bilinear transformation which maps $0, 1, \infty$ of the z-plane in to the points -5, -1, 3 respectively of the w-plane. Find the fixed and critical points of this transformation. [15]

6. (a) Find 'k' such that f(x,y) = $x^3 + 3kxy^2$ is harmonic & find its conjugate.
   (b) Find v,the conjugate harmonic of u=$e^{x^2-y^2}$ cos2xy. Hence find f(z) = u+iv in terms of 'z'. [15]

7. (a) For the function f(z) = $\frac{2z^3+1}{z^2+z}$ find Laurent series expansion in region 2<|z-3|<3
   (b) Explain f(z)= Cos z in Taylor's series about z = πi. [15]

8. (a) Show that J_4 (x) = $\left(\frac{48}{x^3} - \frac{8}{x}\right) J_1(x) + \left(1 - \frac{24}{x^2}\right) J_0(x)$
   (b) Show that $\int_0^\alpha \frac{x^2}{(1+x^4)^3} dx = \frac{5\pi\sqrt{2}}{128}$ [15]

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Code No: R09221801 **R09** **Set No. 3**

II B.Tech II Semester Examinations,April/May 2012
MATHEMATICS - III
Metallurgy And Material Technology

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Find the residue of $\frac{1}{(z-Sinz)}$ at z = 0.
   (b) Evaluate $\int \frac{(z-3)dz}{(z^2+2z+5)}$ where c is $|z+1-i|=2$. [7+8]

2. (a) Prove that $\int_0^\alpha \frac{x^8(1-x^6)dx}{(1+x)^{24}}$ =0 using $\beta - \Gamma$ functions
   (b) Evaluate $\int_0^{\pi/2} \sin^{7/2}\theta \cos^{3/2}\theta d\theta$ [15]

3. (a) Evaluate $\oint_c \frac{z-1}{(z+1)^2(z-2)}dz$ where C: $|z-i|=2$
   (b) Evaluate $\oint_c \frac{z+4}{z^2+2z+5}dz$ where C: $|z+1-i|=2$ [15]

4. (a) Find the Taylor's series expansion of f(z) = $\log\left(\frac{1+z}{1-z}\right)$ about z = 0. Also find the radius of convergence.
   (b) Expand $\frac{7z^2+9z-18}{z^3-9z}$ about $|z-3|>6$ as Laurent's series. Find the region of convergence. [15]

5. (a) Prove that under the transformation w = $\frac{1}{z}$, the images of the lines y=x-1 and y = 0 are the circle $u^2+v^2-u-v=0$ and the line v = 0 respectively.
   (b) In the transformation $z = \frac{i-w}{i+w}$, show that the positive half of the w-plane given by v≥0 corresponds to the circle $|z| \geq 1$ in the z-plane. [8+7]

6. (a) Show that $\int_0^1 x^2 P_{n+1}(x)P_{n-1}(x)dx = \frac{2n(n+1)}{(4n^2-1)(2n+3)}$
   (b) Show that $2P_3(x)+3P_1(x) = 5x^3$ [15]

7. (a) If $\phi(x,y)\&\Psi(x,y)$ satisfy the laplace equations, Show that S+iT is analytic, where
   S = $\frac{\partial\phi}{\partial y} - \frac{\partial\Psi}{\partial x}, T = \frac{\partial\phi}{\partial x} + \frac{\partial\Psi}{\partial y}$.
   (b) Show that the function defined by the equations f(z) = $\begin{cases} u(x,y)+iv(x,y) & if\,(z)\neq 0 \\ 0 & if\,(z)=0 \end{cases}$
   where u(x,y) = $\frac{x^3-y^3}{x^2+y^2}$, v(x,y) =$\frac{x^3+y^3}{x^2+y^2}$ is not analytic at z = 0 eventhough C-R equations are satisfied at that point [15]

8. Find the minimal spanning tree for the following Graph (Figure 4) using Prim's algorithm. [15]

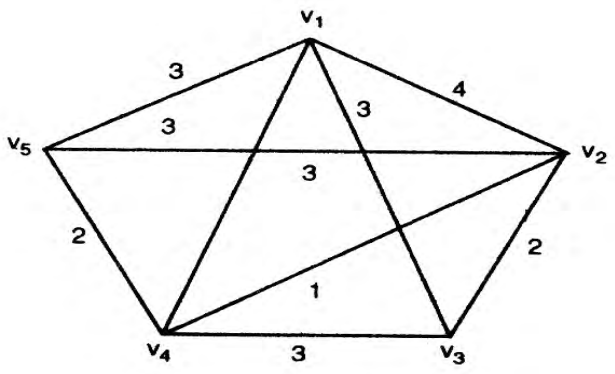

Figure 4:

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