Code No: R05221801 **R05** **Set No. 2**

II B.Tech II Semester Examinations,April/May 2012
MATHEMATICS - III
Metallurgy And Material Technology

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Show that $\int_0^1 x^m (\log x)^n \, dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$ where n is a positive interger and m>-1.
   (b) Show that β(m,n)= $\int_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}} dy$.
   (c) Show that $\int_0^\infty x^4 e^{-x^2} dx = \frac{3\sqrt{\pi}}{8}$. [6+5+5]

2. (a) Evaluate $\int_C \frac{Cos\, z - \sin\, z \;\; dz}{(z+\pi/2)^3}$ with C = | z | = 2 using Cauchy's integral formula.
   (b) Evaluate $\int_{1-i}^{2+i} (2x + 1 + iy)dz$ along (1-i) to (2+i) using Cauchy's integral formula. [8+8]

3. (a) Test for analyticity at the origin for $f(z) = \begin{cases} \frac{x^3 y(y-ix)}{x^6+y^2} & for \quad z \neq 0 \\ 0 & for \quad z = 0 \end{cases}$
   (b) Find all values of z which satisfy (i) e^z = 1+i (ii) sinz =2. [8+8]

4. (a) Expand as a Taylor series in f(z) =$\frac{2z^3+1}{z^2+1}$ about z=1.
   (b) Express f(z) = $\frac{z}{(z-1)\,(z-3)}$ in a series of positive and negative powers of (z-1). [8+8]

5. (a) Prove that $\int x J_0^2 dx = \frac{1}{2} x^2 (J_0^2 + J_1^2) + C$.
   (b) $P_n\,(x)$=$P'_{n+1} - 2x\, P'_n(x) + P'_{n-1}(x)$ [8+8]

6. (a) Find the poles and residue at each pole of the function $\frac{2z+1}{(1-z^4)}$.
   (b) Evaluate $\int_C \frac{\sin z}{z\, \cos\, z} dz$,where C is | z | = π , by residue theorem. [8+8]

7. (a) Find the bilinear transformation which maps the points z=1,–1,0 into the points 1,0,i.
   (b) Show that the straight lines in the z-plane passing through the origin are mapped into straight lines in the w-plane under the transformation w=z^2. [8+8]

8. (a) Show that $\int_0^\pi \frac{Cos 2\theta}{1-2aCos\theta+a^2} = \frac{\pi a^2}{\sqrt{1-a^2}}$, (a^2 < 1) using residue theorem.

(b) Show by the method of contour integration that $\int\limits_0^\infty \frac{Cosmx}{(a^2+x^2)^2} dx = \frac{\pi}{4a^3}(1+ma)e^{-ma}$, (a > 0 , b > 0). [8+8]

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Code No: R05221801 **R05** **Set No. 4**

II B.Tech II Semester Examinations,April/May 2012
MATHEMATICS - III
Metallurgy And Material Technology

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Prove that $\int xJ_0^2 dx = \frac{1}{2}x^2(J_0^2 + J_1^2) + C$.
   (b) $P_n(x)$=$P'_{n+1} - 2x\, P'_n(x) + P'_{n-1}(x)$ [8+8]

2. (a) Show that $\int\limits_0^1 x^m (\log x)^n \, dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$ where n is a positive interger and m>-1.
   (b) Show that β(m,n)= $\int\limits_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}} dy$.
   (c) Show that $\int\limits_0^\infty x^4 e^{-x^2} dx = \frac{3\sqrt{\pi}}{8}$. [6+5+5]

3. (a) Expand as a Taylor series in f(z) =$\frac{2z^3+1}{z^2+1}$ about z=1.
   (b) Express f(z) = $\frac{z}{(z-1)\,(z-3)}$ in a series of positive and negative powers of (z-1). [8+8]

4. (a) Evaluate $\int\limits_C \frac{Cos\, z - \sin\, z \;\; dz}{(z+\pi/2)^3}$ with C = $|\, z\, |$ = 2 using Cauchy's integral formula.
   (b) Evaluate $\int\limits_{1-i}^{2+i} (2x + 1 + iy)dz$ along (1-i) to (2+i) using Cauchy's integral formula. [8+8]

5. (a) Find the poles and residue at each pole of the function $\frac{2z+1}{(1-z^4)}$.
   (b) Evaluate $\int\limits_C \frac{\sin z}{z \cos z} dz$,where C is $|\, z\, | = \pi$, by residue theorem. [8+8]

6. (a) Test for analyticity at the origin for $f(z) = \begin{cases} \frac{x^3 y(y-ix)}{x^6+y^2} & for \quad z \neq 0 \\ 0 & for \quad z = 0 \end{cases}$
   (b) Find all values of z which satisfy (i) e^z = 1+i (ii) sinz =2. [8+8]

7. (a) Find the bilinear transformation which maps the points z=1,–1,0 into the points 1,0,i.
   (b) Show that the straight lines in the z-plane passing through the origin are mapped into straight lines in the w-plane under the transformation w=z^2. [8+8]

8. (a) Show that $\int\limits_0^\pi \frac{Cos 2\theta}{1-2aCos\theta+a^2} = \frac{\pi a^2}{\sqrt{1-a^2}}$, (a^2 < 1) using residue theorem.

(b) Show by the method of contour integration that $\int\limits_0^{\infty} \frac{Cosmx}{(a^2+x^2)^2} dx = \frac{\pi}{4a^3}(1+ma)e^{-ma}$, (a > 0 , b > 0). [8+8]

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Code No: R05221801 **R05** **Set No. 1**

II B.Tech II Semester Examinations,April/May 2012
MATHEMATICS - III
Metallurgy And Material Technology

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Show that $\int\limits_0^\pi \frac{Cos2\theta}{1-2aCos\theta+a^2} = \frac{\pi a^2}{\sqrt{1-a^2}}$, ($a^2 < 1$) using residue theorem.
   (b) Show by the method of contour integration that $\int\limits_0^\infty \frac{Cosmx}{(a^2+x^2)^2}dx = \frac{\pi}{4a^3}(1+ma)e^{-ma}$, (a > 0 , b > 0). [8+8]
2. (a) Prove that $\int xJ_0^2 dx = \frac{1}{2}x^2(J_0^2 + J_1^2) + C$.
   (b) $P_n(x)=P'_{n+1} - 2x\,P'_n(x) + P'_{n-1}(x)$ [8+8]
3. (a) Find the poles and residue at each pole of the function $\frac{2z+1}{(1-z^4)}$.
   (b) Evaluate $\int\limits_C \frac{\sin z}{z\cos z}dz$,where C is $|\,z\,| = \pi$, by residue theorem. [8+8]
4. (a) Evaluate $\int\limits_C \frac{Cos\,z-\sin\,z\ \ dz}{(z+\pi/2)^3}$ with C = $|\,z\,| = 2$ using Cauchy's integral formula.
   (b) Evaluate $\int\limits_{1-i}^{2+i}(2x+1+iy)dz$ along (1-i) to (2+i) using Cauchy's integral formula. [8+8]
5. (a) Show that $\int\limits_0^1 x^m(\log x)^n\,dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$ where n is a positive interger and m>-1.
   (b) Show that β(m,n)= $\int\limits_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}}dy$.
   (c) Show that $\int\limits_0^\infty x^4 e^{-x^2}dx = \frac{3\sqrt{\pi}}{8}$. [6+5+5]
6. (a) Test for analyticity at the origin for $f(z) = \begin{cases} \frac{x^3y(y-ix)}{x^6+y^2} & for\ \ z \neq 0 \\ 0 & for\ \ z = 0 \end{cases}$
   (b) Find all values of z which satisfy (i) e^z = 1+i (ii) sinz =2. [8+8]
7. (a) Expand as a Taylor series in f(z) $=\frac{2z^3+1}{z^2+1}$ about z=1.
   (b) Express f(z) = $\frac{z}{(z-1)\,(z-3)}$ in a series of positive and negative powers of (z-1). [8+8]
8. (a) Find the bilinear transformation which maps the points z=1,–1,0 into the points 1,0,i.

(b) Show that the straight lines in the z-plane passing through the origin are mapped into straight lines in the w-plane under the transformation $w=z^2$. [8+8]

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Code No: R05221801 **R05** **Set No. 3**

II B.Tech II Semester Examinations,April/May 2012
MATHEMATICS - III
Metallurgy And Material Technology

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Show that $\int\limits_0^1 x^m (\log x)^n \, dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$ where n is a positive interger and m>-1.
   (b) Show that β(m,n)= $\int\limits_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}} dy.$
   (c) Show that $\int\limits_0^\infty x^4 e^{-x^2} dx = \frac{3\sqrt{\pi}}{8}.$ [6+5+5]

2. (a) Test for analyticity at the origin for $f(z) = \begin{cases} \frac{x^3 y(y-ix)}{x^6+y^2} & for \quad z \neq 0 \\ 0 & for \quad z = 0 \end{cases}$
   (b) Find all values of z which satisfy (i) e^z = 1+i (ii) sinz =2. [8+8]

3. (a) Show that $\int\limits_0^\pi \frac{Cos 2\theta}{1-2aCos\theta+a^2} = \frac{\pi a^2}{\sqrt{1-a^2}}$, (a^2 < 1) using residue theorem.
   (b) Show by the method of contour integration that $\int\limits_0^\infty \frac{Cos mx}{(a^2+x^2)^2} dx = \frac{\pi}{4a^3}(1 + ma)e^{-ma}$, (a > 0 , b > 0). [8+8]

4. (a) Expand as a Taylor series in f(z) $=\frac{2z^3+1}{z^2+1}$ about z=1.
   (b) Express f(z) = $\frac{z}{(z-1)\,(z-3)}$ in a series of positive and negative powers of (z-1). [8+8]

5. (a) Evaluate $\int\limits_C \frac{Cos\, z - \sin\, z \;\; dz}{(z+\pi/2)^3}$ with C = $|\, z \,|$ = 2 using Cauchy's integral formula.
   (b) Evaluate $\int\limits_{1-i}^{2+i} (2x + 1 + iy) dz$ along (1-i) to (2+i) using Cauchy's integral formula. [8+8]

6. (a) Find the poles and residue at each pole of the function $\frac{2z+1}{(1-z^4)}$.
   (b) Evaluate $\int\limits_C \frac{\sin z}{z \cos z} dz$,where C is $|\, z \,| = \pi$, by residue theorem. [8+8]

7. (a) Prove that $\int x J_0^2 dx = \frac{1}{2} x^2 (J_0^2 + J_1^2) + C.$
   (b) $P_n\,(x)$=$P'_{n+1} - 2x\, P'_n(x) + P'_{n-1}(x)$ [8+8]

8. (a) Find the bilinear transformation which maps the points z=1,–1,0 into the points 1,0,i.

(b) Show that the straight lines in the z-plane passing through the origin are mapped into straight lines in the w-plane under the transformation $w=z^2$. [8+8]

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