

**II B.Tech II Semester Examinations, April/May 2012**  
**MATHEMATICS - III**  
**Metallurgy And Material Technology**

Time: 3 hours

Max Marks: 80

**Answer any FIVE Questions**  
**All Questions carry equal marks**

\*\*\*\*\*

1. (a) Evaluate  $\int_{1-i}^{2+i} (2x + iy + 1)dz$  along the two paths :
  - i.  $x = t+1, y = 2t^2-1$
  - ii. the straight line joining  $(1-i)$  and  $(2+i)$  .
 (b) Use Cauchy's integral formula to calculate :  
 $\int_C \frac{e^{-z}}{(z+1)} dz$  where C is the circle  $|z| = 2$  [10+6]
2. (a) Express  $f(x) = 5x^3 + x$  in terms of Legendre's polynomials .  
 (b) Prove that  $\int_{-1}^1 x^m P_n(x) dx = 0, \text{ if } m, n \text{ are positive integers and } m < n. \text{ [8+8]}$
3. Evaluate  $\int_C \frac{f'(z)}{f(z)} dz$  by using Argument principle where C is a simple closed curve C and  $f(z) = z^5 - 8z^2i + 2z - 3 + 5i.$  [16]
4. (a) Prove that the function  $f(z) = \sqrt{|xy|}$  is not analytic at the origin even though the C - R equations are satisfied there at.  
 (b) Find the analytic function whose real part is  $y / (x^2 + y^2).$  [8+8]
5. (a) Expand  $f(z) = \frac{z-1}{z^2}$  in a Taylor's series in power of  $(z - 1)$  and determine the region of convergence.  
 (b) Determine the poles of the function  $f(z) = \frac{e^z}{z^2 + \pi^2}$  [8+8]
6. Show that  $\int_0^{2\pi} \frac{d\theta}{a+b \sin \theta} = \int_0^{2\pi} \frac{d\theta}{a+b \cos \theta} = \frac{2\pi}{\sqrt{a^2-b^2}}, a > b > 0$  using residue theorem. [16]
7. (a) Prove that the real part of the principal value of  $i^{\log (1+i)}$  is  $e^{-\frac{\pi^2}{8}} \cos \left(\frac{\pi}{4} \log 2\right)$   
 (b) If  $\tan(x + iy) = \sin(A + iB).$  Prove that  $\frac{\sin 2x}{\sinh 2y} = \frac{\tan A}{\tanh B}$  [8+8]
8. (a) Prove that the transformation  $w = \frac{1}{z}$  maps every straight line or circle on to a circle or straight line.  
 (b) Define bilinear transformation. Prove that the bilinear transformation is conformal. [8+8]

\*\*\*\*\*

**II B.Tech II Semester Examinations, April/May 2012**  
**MATHEMATICS - III**  
**Metallurgy And Material Technology**

Time: 3 hours

Max Marks: 80

**Answer any FIVE Questions**  
**All Questions carry equal marks**

\*\*\*\*\*

1. (a) Evaluate  $\int_{1-i}^{2+i} (2x + iy + 1)dz$  along the two paths :
  - i.  $x = t+1, y = 2t^2-1$
  - ii. the straight line joining  $(1-i)$  and  $(2+i)$  .
- (b) Use Cauchy's integral formula to calculate :  
 $\int_C \frac{e^{-z}}{(z+1)} dz$  where C is the circle  $|z| = 2$  [10+6]
2. (a) Prove that the function  $f(z) = \sqrt{|xy|}$  is not analytic at the origin even though the C - R equations are satisfied there at.
- (b) Find the analytic function whose real part is  $y / (x^2 + y^2)$ . [8+8]
3. (a) Express  $f(x) = 5x^3 + x$  in terms of Legendre's polynomials .
- (b) Prove that  $\int_{-1}^1 x^m P_n(x) dx = 0, if m, n$  are positive integers and  $m < n$ . [8+8]
4. (a) Prove that the transformation  $w = \frac{1}{z}$  maps every straight line or circle on to a circle or straight line.
- (b) Define bilinear transformation. Prove that the bilinear transformation is conformal. [8+8]
5. Evaluate  $\int_C \frac{f'(z)}{f(z)} dz$  by using Argument principle where C is a simple closed curve C and  $f(z) = z^5 - 8z^2i + 2z - 3 + 5i$ . [16]
6. Show that  $\int_0^{2\pi} \frac{d\theta}{a+b \sin \theta} = \int_0^{2\pi} \frac{d\theta}{a+b \cos \theta} = \frac{2\pi}{\sqrt{a^2-b^2}}, a > b > 0$  using residue theorem. [16]
7. (a) Expand  $f(z) = \frac{z-1}{z^2}$  in a Taylor's series in power of  $(z - 1)$  and determine the region of convergence.
- (b) Determine the poles of the function  $f(z) = \frac{e^z}{z^2 + \pi^2}$  [8+8]
8. (a) Prove that the real part of the principal value of  $i^{\log(1+i)}$  is  $e^{-\frac{\pi^2}{8}} \cos\left(\frac{\pi}{4} \log 2\right)$
- (b) If  $\tan(x + iy) = \sin(A + iB)$ . Prove that  $\frac{\sin 2x}{\sinh 2y} = \frac{\tan A}{\tanh B}$  [8+8]

\*\*\*\*\*

## II B.Tech II Semester Examinations, April/May 2012

## MATHEMATICS - III

## Metallurgy And Material Technology

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions

All Questions carry equal marks

\*\*\*\*\*

1. (a) Prove that the transformation  $w = \frac{1}{z}$  maps every straight line or circle on to a circle or straight line.  
 (b) Define bilinear transformation. Prove that the bilinear transformation is conformal. [8+8]
2. (a) Expand  $f(z) = \frac{z-1}{z^2}$  in a Taylor's series in power of  $(z - 1)$  and determine the region of convergence.  
 (b) Determine the poles of the function  $f(z) = \frac{e^z}{z^2 + \pi^2}$  [8+8]
3. Show that  $\int_0^{2\pi} \frac{d\theta}{a+b \sin \theta} = \int_0^{2\pi} \frac{d\theta}{a+b \cos \theta} = \frac{2\pi}{\sqrt{a^2 - b^2}}$ ,  $a > b > 0$  using residue theorem. [16]
4. (a) Evaluate  $\int_{1-i}^{2+i} (2x + iy + 1) dz$  along the two paths :  
 i.  $x = t+1, y = 2t^2-1$   
 ii. the straight line joining  $(1-i)$  and  $(2+i)$  .  
 (b) Use Cauchy's integral formula to calculate :  
 $\int_C \frac{e^{-z}}{(z+1)} dz$  where C is the circle  $|z| = 2$  [10+6]
5. (a) Prove that the real part of the principal value of  $i^{\log(1+i)}$  is  $e^{-\frac{\pi^2}{8}} \cos\left(\frac{\pi}{4} \log 2\right)$   
 (b) If  $\tan(x + iy) = \sin(A + iB)$ . Prove that  $\frac{\sin 2x}{\sinh 2y} = \frac{\tan A}{\tanh B}$  [8+8]
6. (a) Prove that the function  $f(z) = \sqrt{|xy|}$  is not analytic at the origin even though the C - R equations are satisfied there at.  
 (b) Find the analytic function whose real part is  $y / (x^2 + y^2)$ . [8+8]
7. (a) Express  $f(x) = 5x^3 + x$  in terms of Legendre's polynomials .  
 (b) Prove that  $\int_{-1}^1 x^m P_n(x) dx = 0$ , if  $m, n$  are positive integers and  $m < n$ . [8+8]
8. Evaluate  $\int_C \frac{f'(z)}{f(z)} dz$  by using Argument principle where C is a simple closed curve C and  $f(z) = z^5 - 8z^2i + 2z - 3 + 5i$ . [16]

\*\*\*\*\*

## II B.Tech II Semester Examinations, April/May 2012

## MATHEMATICS - III

## Metallurgy And Material Technology

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions  
All Questions carry equal marks

\*\*\*\*\*

1. (a) Evaluate  $\int_{1-i}^{2+i} (2x + iy + 1)dz$  along the two paths :
  - i.  $x = t+1, y = 2t^2-1$
  - ii. the straight line joining  $(1-i)$  and  $(2+i)$  .
 (b) Use Cauchy's integral formula to calculate :  
 $\int_C \frac{e^{-z}}{(z+1)} dz$  where C is the circle  $|z| = 2$  [10+6]
2. Evaluate  $\int_C \frac{f'(z)}{f(z)} dz$  by using Argument principle where C is a simple closed curve C and  $f(z) = z^5 - 8z^2i + 2z - 3 + 5i$ . [16]
3. (a) Prove that the real part of the principal value of  $i^{\log(1+i)}$  is  $e^{-\frac{\pi^2}{8}} \cos\left(\frac{\pi}{4} \log 2\right)$   
 (b) If  $\tan(x + iy) = \sin(A + iB)$ . Prove that  $\frac{\sin 2x}{\sinh 2y} = \frac{\tan A}{\tanh B}$  [8+8]
4. (a) Prove that the function  $f(z) = \sqrt{|xy|}$  is not analytic at the origin even though the C - R equations are satisfied there at.  
 (b) Find the analytic function whose real part is  $y / (x^2 + y^2)$ . [8+8]
5. (a) Expand  $f(z) = \frac{z-1}{z^2}$  in a Taylor's series in power of  $(z - 1)$  and determine the region of convergence.  
 (b) Determine the poles of the function  $f(z) = \frac{e^z}{z^2 + \pi^2}$  [8+8]
6. (a) Express  $f(x) = 5x^3 + x$  in terms of Legendre's polynomials .  
 (b) Prove that  $\int_{-1}^1 x^m P_n(x) dx = 0, \text{ if } m, n \text{ are positive integers and } m < n$ . [8+8]
7. (a) Prove that the transformation  $w = \frac{1}{z}$  maps every straight line or circle on to a circle or straight line.  
 (b) Define bilinear transformation. Prove that the bilinear transformation is conformal. [8+8]
8. Show that  $\int_0^{2\pi} \frac{d\theta}{a+b \sin \theta} = \int_0^{2\pi} \frac{d\theta}{a+b \cos \theta} = \frac{2\pi}{\sqrt{a^2 - b^2}}, a > b > 0$  using residue theorem. [16]

\*\*\*\*\*