**R15**

Code No: 123BT

**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD**
**B.Tech II Year I Semester Examinations, December - 2019**
**PROBABILITY THEORY AND STOCHASTIC PROCESSES**
**(Electronics and Communication Engineering)**

**Time: 3 Hours** **Max. Marks: 75**

**Note:** This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit.
Each question carries 10 marks and may have a, b, c as sub questions.

**PART- A** **(25 Marks)**

1.a) Define the term Independent events. State the conditions for independence of any three events A, B and C. [2]
b) In a single throw of two dice, what is the probability of obtaining a sum of at least 10? [3]
c) Differentiate conditional Event and conditional Density. [2]
d) What is the distribution function of mixed random variable? [3]
e) Define Joint Central Moments and where it is applicable. [2]
f) Define and mention the purpose of central limit theorem. [3]
g) Define Cross-Correlation Function and its Properties. [2]
h) What is mean by strict-sense stationarity and derive condition for it? [3]
i) Define Cross-Power Spectral Density and mention its purpose. [2]
j) Define the terms (i)Power spectrum and (ii) Average power. What is need of their measurement. [3]

**PART-B** **(50 Marks)**

2.a) If A and B are any events, not necessarily mutually exclusive events, derive an expression for probability of A Union B. When A and B are mutually exclusive, what happens to the above expression derived?
b) State and prove Bayes theorem of probability. [5+5]

**OR**

3.a) Define the following terms:
i) joint probability
ii) conditional probability
iii) total probability
b) Consider the experiment of tossing four coins. The random variable x is associated with the number of tails showing. Compute and sketch the CDF of x. [5+5]

4.a) If the mean and variance of the binomial distribution are 6 and 1.5 respectively. Find $E[X + P(X \geq 3)]$
b) For a function $Y=(X - mx)/\sigma x$, prove that mean is zero & variance is 1. [5+5]

**OR**

5.a) An analog signal received at the detector (measured in micro volts) may be modeled as a Gaussian random variable N (200,256) at a fixed point in time. What is the probability that the signal will exceed 240 μvs. What is the probability that the signal is larger than 240 μV, given that it is larger than 210 μVs?
b) State and prove any four properties of characteristic function. [5+5]

6.a) Let X and Y be independent random variables each N(0,1). Find the mean and variance of $Z = \sqrt{X^2 + Y^2}$

b) For two zero mean Gaussian random variables X and Y show that their joint characteristic function is $\varphi_{XY}(\omega_1,\omega_2) = \exp\left\{-\frac{1}{2}\left[\sigma_x^2\omega_1^2 + 2\sigma_x\sigma_y\omega_1\omega_2 + \sigma_y^2\omega_2^2\right]\right\}$ [5+5]

**OR**

7.a) Random variables Z and W are defined by. Z= X+aY W= X-aY. Where 'a' is a real number. Determine 'a' such that Z and W are orthogonal.

b) Joint probabilities of two random variables X and Y are given in Figure 1. [5+5]

| Y \ X | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 0.2 | 0.1 | 0.2 |
| 2 | 0.15 | 0.2 | 0.15 |

Find out
(i) Joint and marginal distribution functions and plot.
(ii) Joint and marginal density functions and plot. [5+5]

8.a) A random process consists of three sample functions $x(t, s_1) = 2$, $x(t,s_2) = 2\cos t$ and $x(t,s_3) = 3 \sin t$ each occurring with equal probability. Is the process stationary in any sense?

b) Let N(t) be a zero mean wide sense stationary noise process for which $R_{NN}(\tau) = (No/2)\,\delta(t)$ where $No > 0$ is a finite constant. Determine whether N(t) is mean ergodic. [5+5]

**OR**

9.a) Given the auto correlation function for a stationary ergodic process with no periodic components is $R_{XX}(\tau) = 25 + 4/(1 + 6\tau^2)$ Find mean and variance of process X(t).

b) Consider a random process $X(t) = \cos(\omega t + \theta)$ where $\omega$ is a real constant and $\theta$ is a uniform random variable in $(0, \pi/2)$. Show that X(t) is not a WSS process. Also find the average power in the process. [5+5]

10.a) A WSS noise process N(t) has ACF $R_{NN}(\tau) = Pe^{-3|\tau|}$. Find PSD and plot both ACF and PSD.

b) The auto correlation function of an periodic random process is $R_{xx}(\tau) = \exp\left[\frac{-x^2}{2\sigma^2}\right]$ find the PSD and average power of the signal? [5+5]

OR

11.a) Derive the relation between PSDs of input and output random process of an LTI system.

b) Find the ACF of the following PSDs
i) $S_{xx}(\omega) = (157 + 12\omega^2) / (16+\omega^2)(9+\omega^2)$
ii) $S_{xx}(\omega) = 8 /(9+ \omega^2)^2$ [5+5]

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